Correcting Sorted Sequences in a Single Hop Radio Network

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Model of computation

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- broadcasting/listening in a single time slot requires a unit of energetic cost
- memory of single station limited (constant number of variables storing either keys or $\lceil \lg_2 n \rceil$-bit integers).

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Energetic cost of the algorithm: the maximal energy dissipated by a single station.

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- The sequence of the *old keys is sorted* (i.e. each station knows the position of its old key in the sorted sequence).
- We want to sort the sequence of the *new keys* (i.e. each station has to learn the index of its new key in the sorted sequence of the new keys).

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- (SOFSEM 2006) simple sorting based on (moderately) balanced merging
  - Time: $O(n \log n)$
  - Energy: $O(\log^2 n)$
  - low constants under O

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k is not fixed nor limited. The algorithm adapts itself to arbitrary $k \leq n$.

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4. `final-merge`: Merging the b-sequence with the a-sequence.

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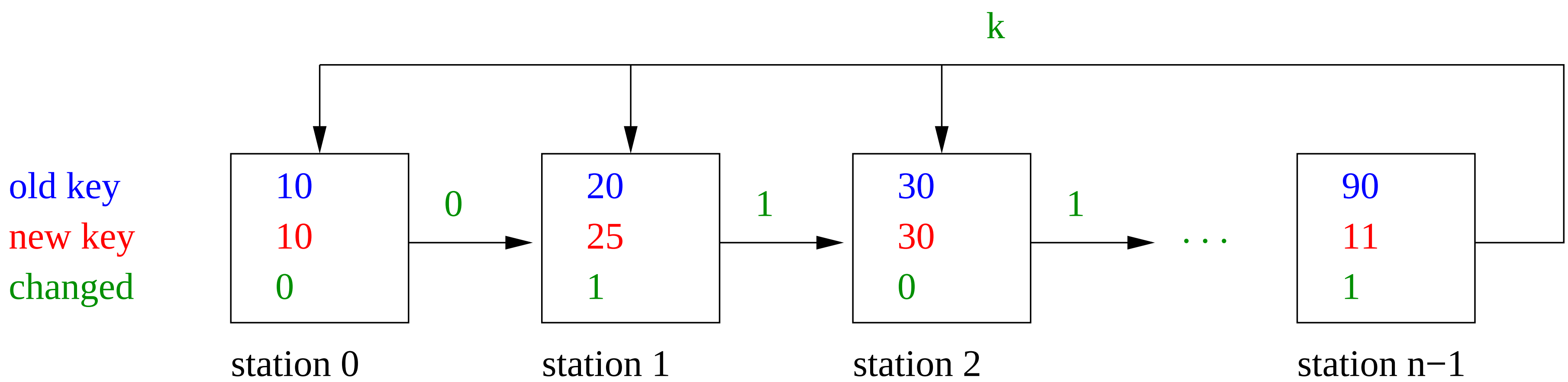
Time: n

Energetic cost: 3

split-and-count



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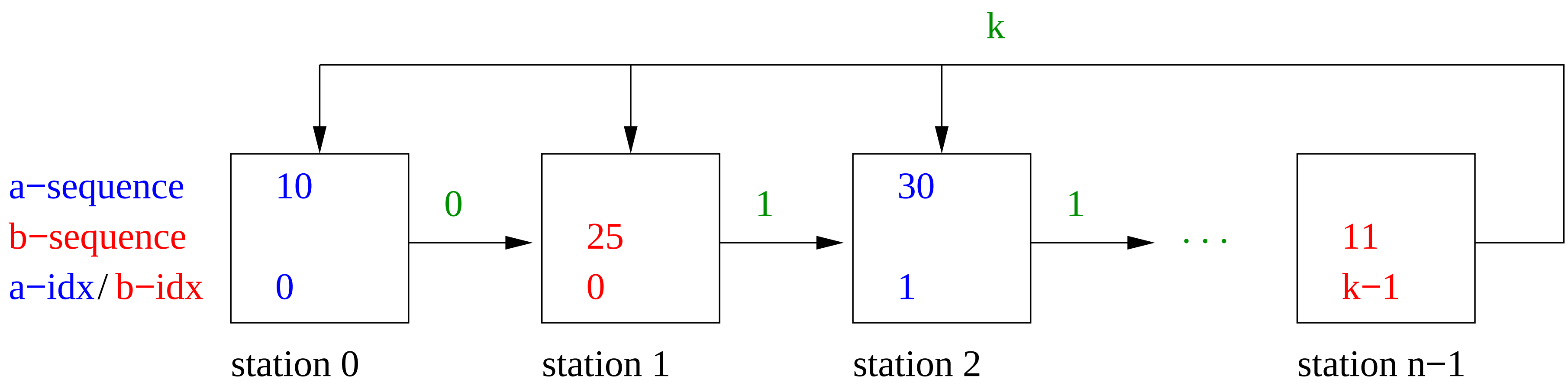

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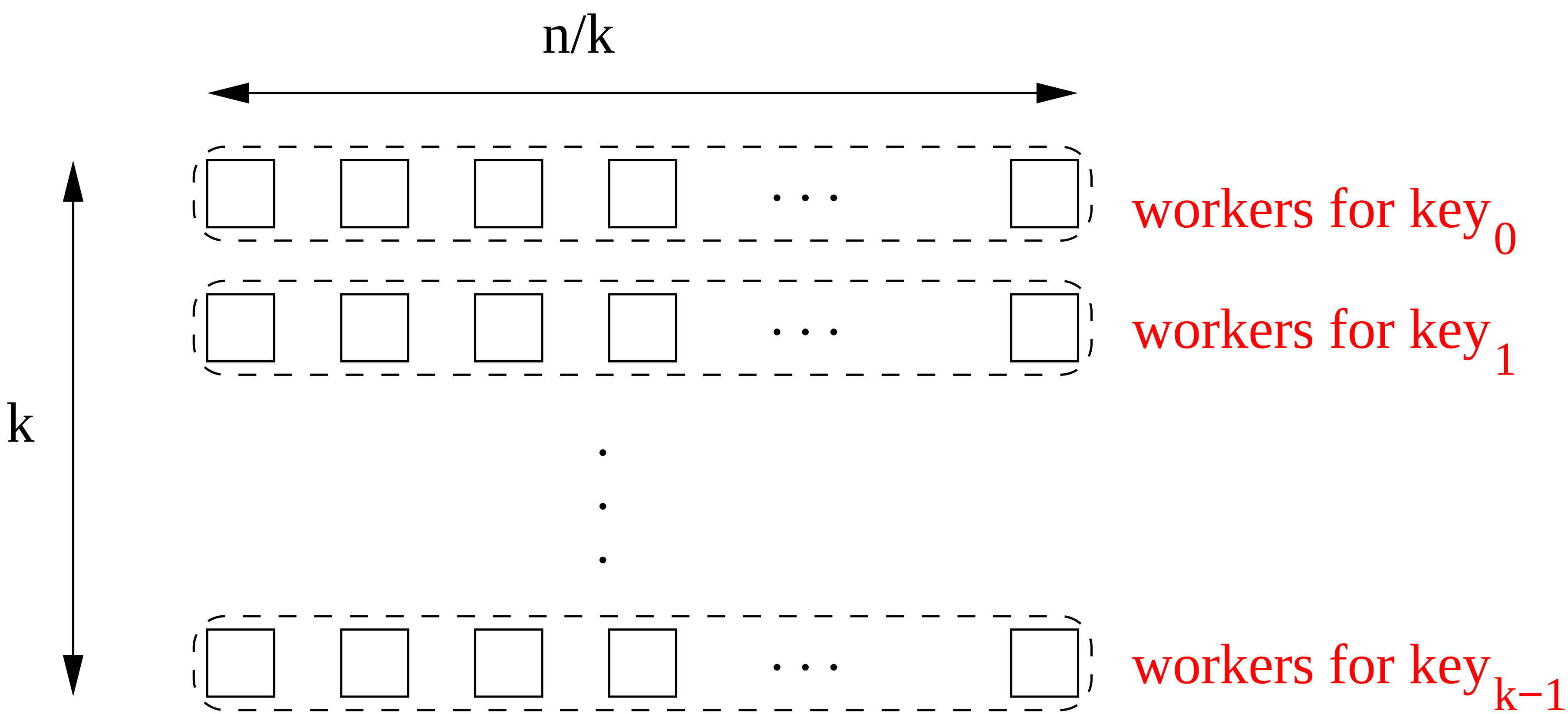
assign-workers

Let key_i denote the ith b-key. All stations know k. They are arranged in the following matrix:



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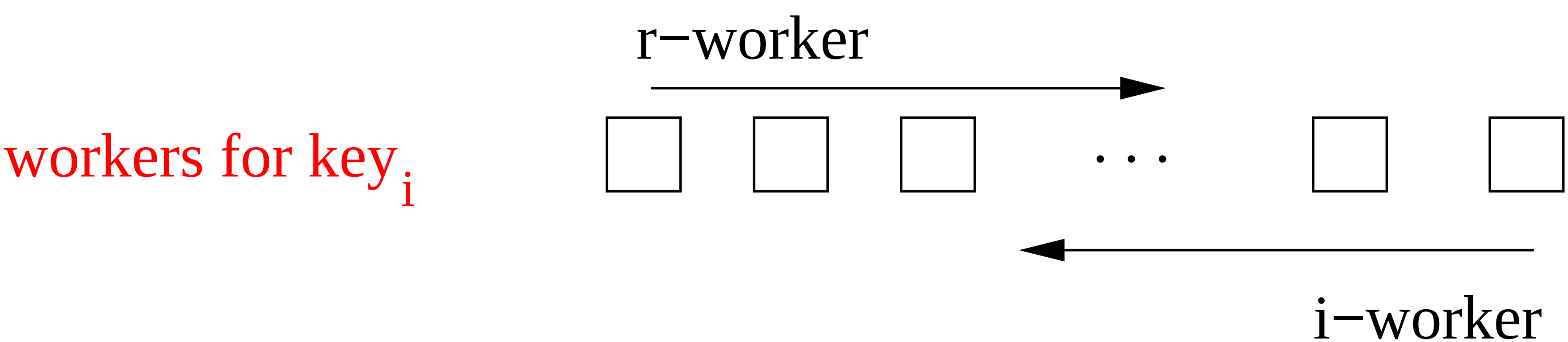
- For $0 \leq i \leq k-1$,
  - the *first* worker for key_i becomes the current *r-worker* (*rank-worker*).
  - the *last* worker for key_i becomes the current *i-worker* (*index-worker*).



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Time: k

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Time: k

Energetic cost: 2

sort

The b-sequence is sorted by a (balanced) merge-sort:

begin

$m \leftarrow 1$;

while $m < k$ **do**

merge all pairs of subsequences of length m;

$m \leftarrow 2 \cdot m$;

end

merging

To merge two sorted sequences, each key from one sequence has to learn its rank in the other sequence. Then it can compute its index in the merged sequence.

procedure `merge`(seq_1, seq_2)
begin
  `rank`(seq_1, seq_2);
  `rank`(seq_2, seq_1);
end

ranking

Ranking seq_1 in seq_2:

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- For each key of seq_i, its current i-worker knows its index in the *sorted* seq_i.

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Ranking seq_1 in seq_2:

- For each key of seq_i, its current i-worker knows its index in the *sorted* seq_i.
- The sorted sequence seq_2, permuted by a special permutation (bso), is transmitted by the i-workers of seq_2. (Each i-worker transmits once.)

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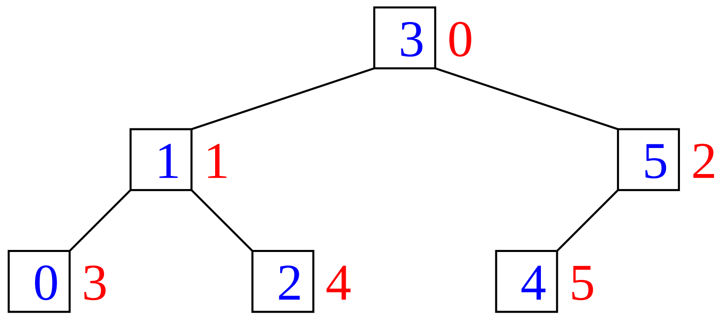
  (Each i-worker transmits once.)
- During these transmissions, for each key of seq_1, some its r-workers are used to compute its rank in seq_2.
  (Each r-worker uses constant energy.)

Ranking: Binary tree T_m, bso

Let m be the length of seq_2. The elements of seq_2 are arranged in the following tree:



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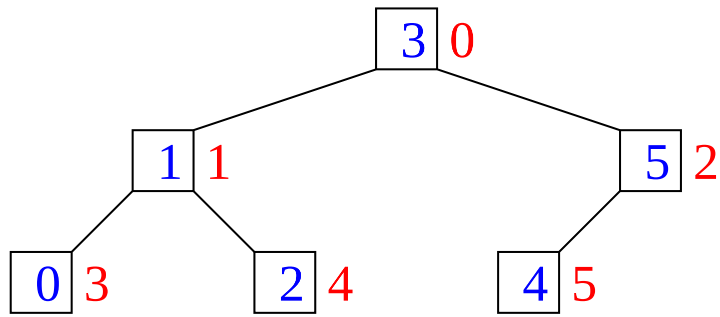

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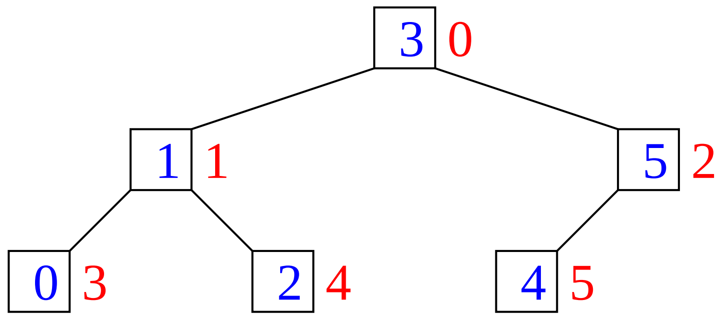

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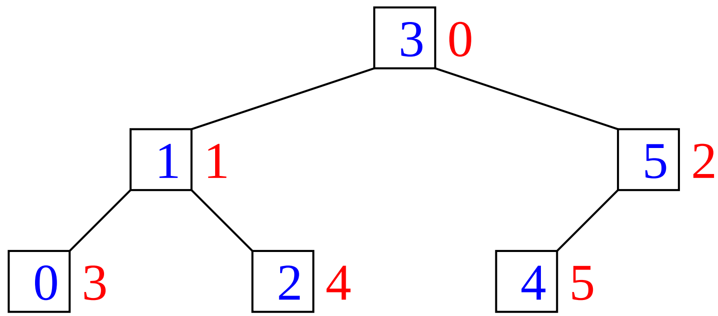

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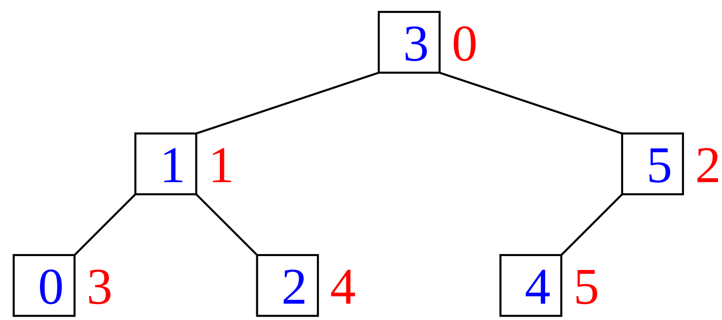

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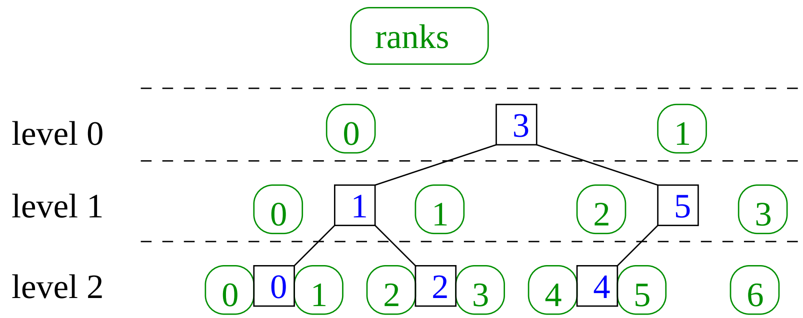
- the ith element of the sorted seq_2 is transmitted as the tth, where $t = bso_m(i)$.

Ranking: Layers of T_m



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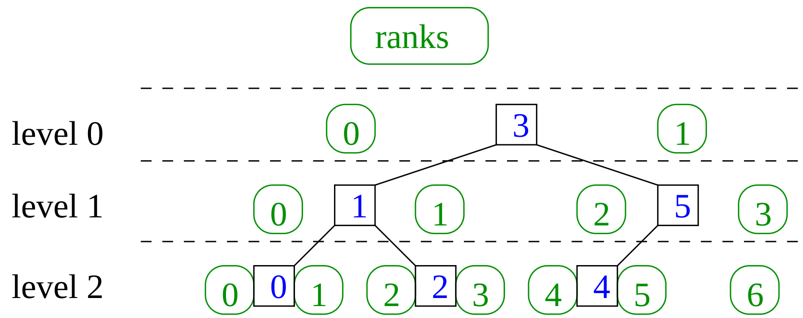

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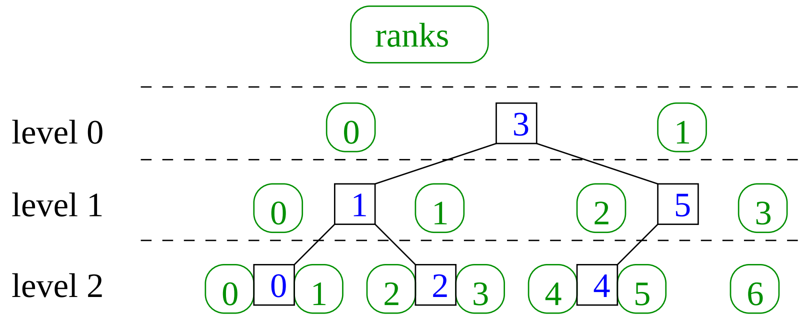

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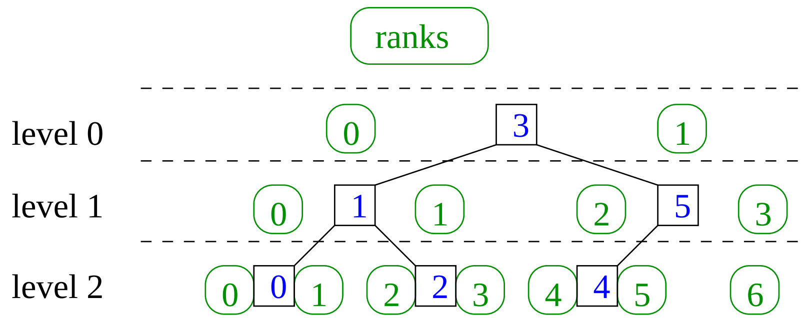

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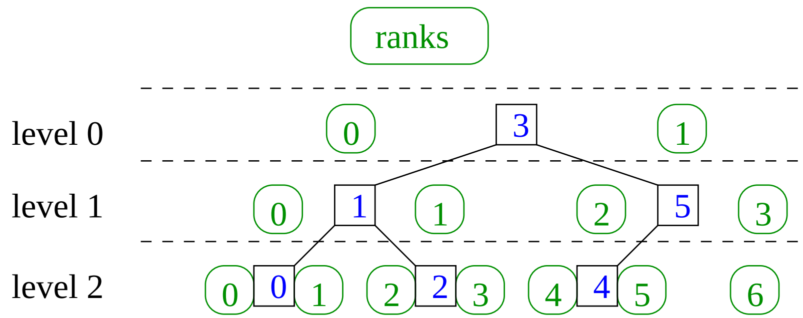

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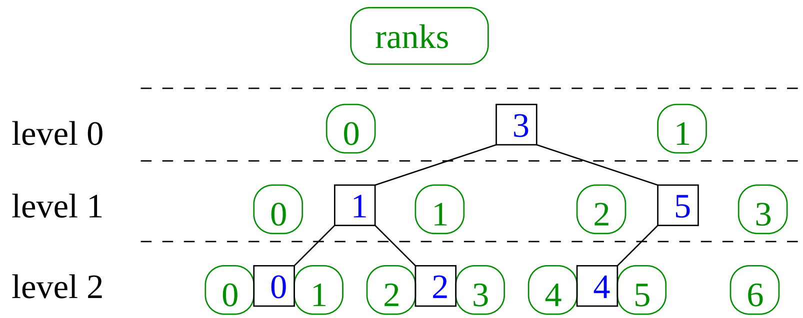

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- between the the levels, the current r-workers of seq_1 transfer the ranks to the next r-workers.

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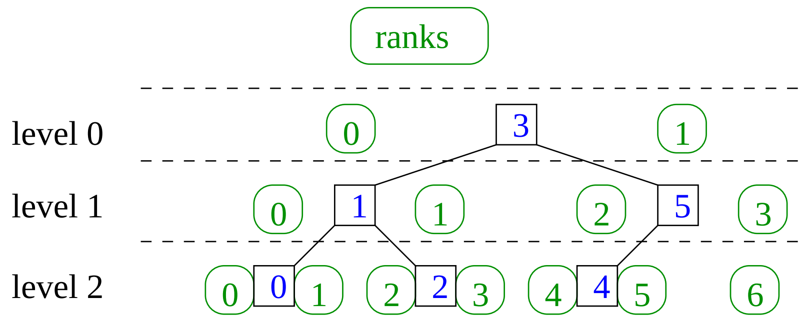

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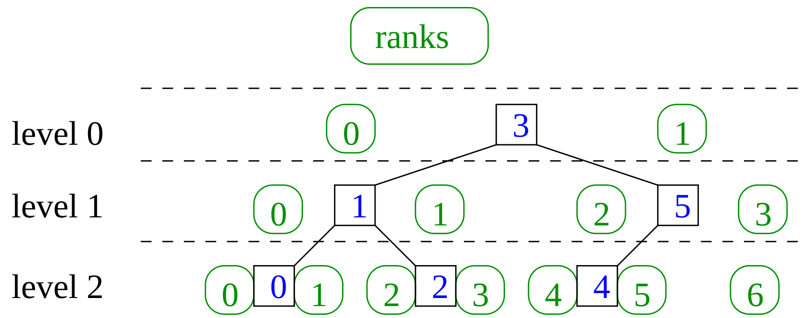

- After the last level, each r-worker of seq_1 sends the rank to the i-worker which computes the index of its key in the sequence merged from seq_1 and seq_2. (Procedure `send-ranks-to-indexes`.)

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- `rank`(b-sequence, a-sequence)   (also blanced!)

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Now each owner of b-key knows:

- its index in the sorted output
- its index in the sorted b-sequence
- its rank in the a-sequence

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Phase 2: Computing output index for each a-key:

- In the sorted b-sequence, each b-key informs its predecessor about its rank in the a-sequence. (Each *last* b-key with given rank becomes aware of this fact.)

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- For $0 \leq t \leq n - k - 1$, in time slot t, the last b-key from the sorted b-sequence with the rank t (if exists) informs the tth a-key from a-sequence about its index in the sorted b-sequence. (a *displacement* of this a-key).

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- For $0 \leq t \leq n-k-2$, in time slot t, the $(t+1)$st a-key that did not receive its displacement from the b-sequence, receives the displacement from its predecessor in a-sequence.

final-merge

Phase 2: Computing output index for each a-key:

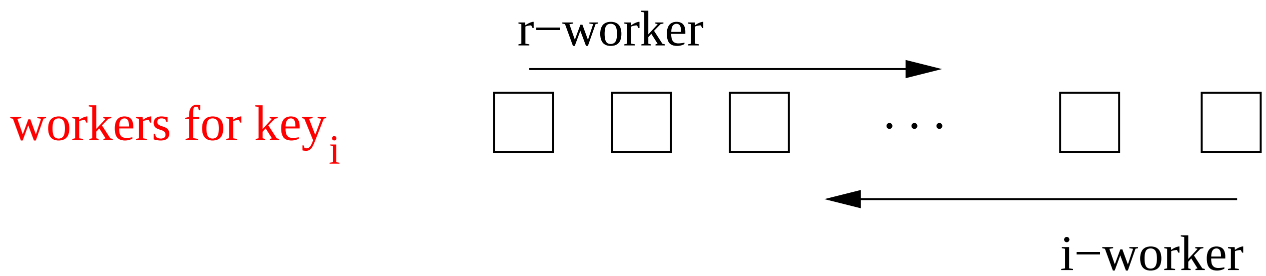
- In the sorted b-sequence, each b-key informs its predecessor about its rank in the a-sequence. (Each *last* b-key with given rank becomes aware of this fact.)
- For $0 \leq t \leq n-k-1$, in time slot t, the last b-key from the sorted b-sequence with the rank t (if exists) informs the tth a-key from a-sequence about its index in the sorted b-sequence. (a *displacement* of this a-key).
- For $0 \leq t \leq n-k-2$, in time slot t, the $(t+1)$st a-key that did not receive its displacement from the b-sequence, receives the displacement from its predecessor in a-sequence.
- Each a-key adds its displacement to its index in a-sequence to get its index in the *sorted sequence of all keys*.

`final-merge` – complexity

Time: $O(n)$

Energetic cost: $O(1) +$ the cost of `rank`

Balancing the energy in `sort` and `final-merge`



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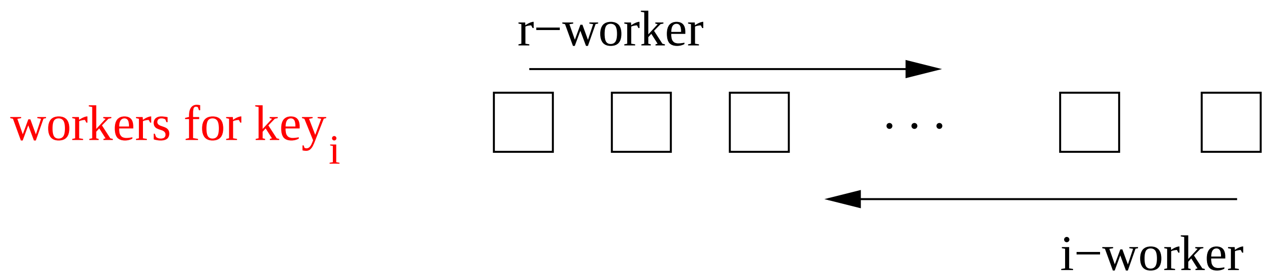
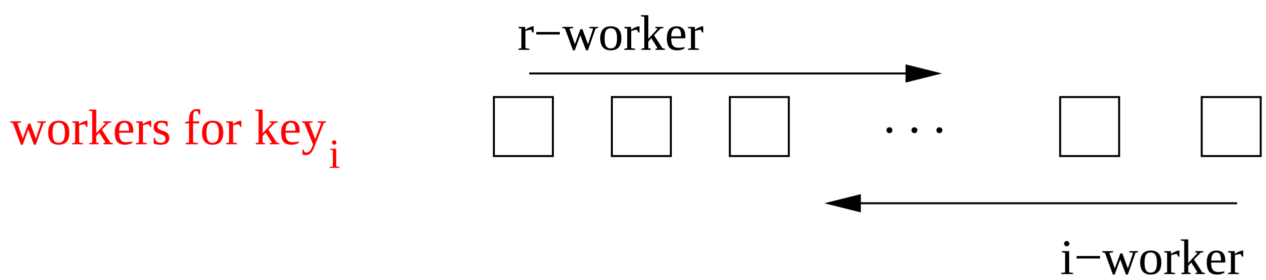


- After each `merge`(seq_1, seq_2) , for each key of seq_1 and seq_2, the task of i-worker is transfered to the previous (modulo $\lfloor n/k \rfloor$) worker.(For each key – at most $\lceil \lg k \rceil$ such transfers.)

Balancing the energy in `sort` and `final-merge`



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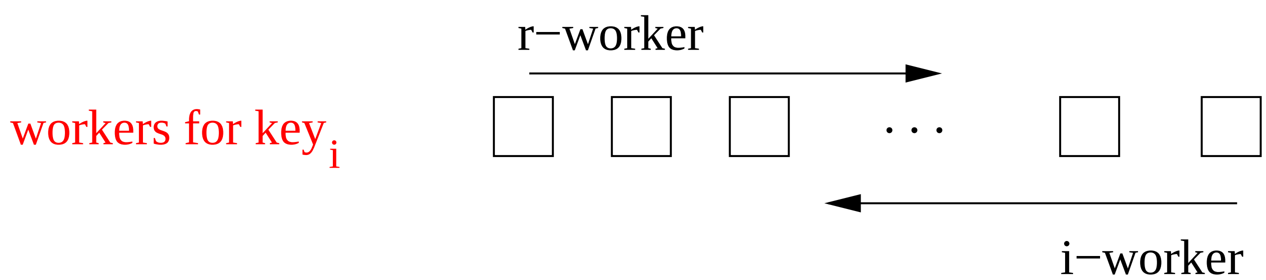
- After each level of `rank`(seq_1, seq_2), for each key of seq_1, the task of r-worker is transfered to the next (modulo $\lfloor n/k \rfloor$) worker.(For each key – at most $\frac{(\lceil \lg k \rceil + 1)(\lceil \lg k \rceil + 2)}{2}$ such transfers.)

Balancing the energy in `sort` and `final-merge`



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- The energetic cost of each transfer is constant.

Final remarks

- Robustness to interferences: How to correct sequences in the model, where each message is received with probability $p < 1$?

  Algorithm for sorting has been proposed on ADHOC-NOW 2008.

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  Algorithm for sorting has been proposed on ADHOC-NOW 2008.

- Simulation in Java available at:

  `http://www.im.pwr.wroc.pl/~kik/CorrectionRN.java`

THE END

THANK YOU!