Available online at www.sciencedirect.com

SCIENCE DIRECT®

Operations Research Letters 31 (2003) 53–59

Operations Research Letters

www.elsevier.com/locate/dsw

On the hardness of evaluating criticality of activities in a planar network with duration intervals

Stefan Chanas[a,*], Paweł Zieliński[b]

[a]Institute of Industrial Engineering and Management, Wrocław University of Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

[b]Institute of Mathematics, Technical University of Opole, Luboszycka 3, 45-036 Opole, Poland

Received 19 December 2001; received in revised form 27 May 2002; accepted 30 May 2002

Abstract

Complexity results for problems of evaluating the criticality of activities in planar networks with duration time intervals are presented. We show that the problems of asserting whether an activity is possibly critical, and of computing bounds on the float of an activity in these networks are NP-complete and NP-hard, respectively.

Keywords: Project management and scheduling; Complexity; Data intervals

1. Introduction

The critical path method (CPM), worked out by Kelly and Walker [5] in 1959, is one of the most frequently used tools in Operations Research. It is applied to the analysis of complex projects from the point of view of the planning and control of their realization in time. The essence of the CPM is the representation of the project by an activity network, where activities with given duration times are related to each other by means of precedence constraints. The identification, in such a network, of the so-called critical activities, i.e. the activities which, under the assumption of minimum project duration, have no time float for their execution and must be started and completed on exactly determined moments, is an important task in practice.

When the durations of activities are precisely known, all of the critical activities are easy to identify in the network by means of the CPM. This problem reduces to determining the longest or critical paths in an acyclic activity network. In case of ill-known activity duration times, the problem becomes more complicated even if their estimations are modeled by intervals containing possible duration times. Such a situation is natural, since many of the activities may be executed in the project for the first time. Strangely enough, this problem, with such a simple form of uncertainty representation for activity durations, has not been analyzed in-depth in the literature so far. Recently, the authors have studied the *possible criticality* of paths and activities in networks with duration time intervals. A path (an activity) is possibly critical in a network only when there exists a set of exact values of durations, such that the path (the activity) is

* Corresponding author. Tel.: +48-71-229-819; fax: +48-71-320-3432.

E-mail addresses: chanas@ioz.pwr.wroc.pl (S. Chanas), pziel@po.opole.pl (P. Zieliński).

0167-6377/03/$ - see front matter © 2002 Elsevier Science B.V. All rights reserved.
PII: S0167-6377(02)00174-8

critical in the usual sense in the network, in which the time intervals have been replaced by the exact times. In [1,2], it has been proved that the problem of evaluating the possible criticality of an activity is probably intractable (NP-complete in the strong sense) for general networks. At the same time, Fargier et al. [3] provided polynomial algorithms for this problem and some related ones, such as the problems of computing bounds on the float and on the latest starting time of an activity, for the case of series-parallel networks.

In this paper we show that the problems of evaluating the possible criticality of an activity and of computing bounds on the float of an activity are probably intractable even when a network is restricted to be planar and regular of degree three, exhausting in this way the analysis of the problems from the point of view of their computational complexity.

2. Preliminaries

A network $S = \langle V, A \rangle$ being a project activity-on-arc model, is given. V is the set of nodes (events), $|V| = n$, and A is the set of arcs (activities), $|A| = m$. The network S is a directed, connected and acyclic graph. The set $V = \{1, 2, \dots, n\}$ is labeled in such a way that $i < j$ for each activity $(i,j) \in A$. Weights of the arcs (activity durations) $(i,j) \in A$ are to be chosen from intervals $T_{ij} = [\underline{t}_{ij}, \bar{t}_{ij}]$, $\underline{t}_{ij} \geqslant 0$, two nodes 1 and n are distinguished as the initial and final node, respectively.

Let $\mathscr{T}$ be a realization of the activity durations $t_{ij} \in T_{ij}$, $(i,j) \in A$, in S. We use $t_{ij}(\mathscr{T})$ to denote the duration of activity (i,j), $(i,j) \in A$, in realization $\mathscr{T}$. We denote the set of all paths in S from node 1 to node n by P.

Let us introduce the notions of possible criticality of activities and paths in the network S.

Definition 1. An activity $(k,l) \in A$ (respectively a path $p \in P$) is possibly critical in S if there exists a realization of times $\mathscr{T}$ such that (k,l) (respectively p) is critical in S in the usual sense, after replacing the time intervals T_{ij} by exact values $t_{ij}(\mathscr{T})$, $(i,j) \in A$.

Similar concepts are proposed in [6] for the spanning tree problem. The terms weak tree, weak edge are used.

In view of the above definition we define the problem PAPC: *assert whether a given activity $(k,l) \in A$ is possibly critical in S.*

The following statement is obvious. It results from the definition given previously.

Statement 1. *An activity $(k,l) \in A$ is possibly critical in S if and only if it belongs to some possibly critical path $p \in P$.*

Fargier et al. [3] studied the criticality of activities from the point of view of floats. They considered the problem of determining the interval F_{kl} of possible values of the float f_{kl} for a given activity $(k,l) \in A$, i.e. the interval $F_{kl} = [\underline{f}_{kl}, \bar{f}_{kl}]$ formed by the $\underline{f}_{kl} = \min f_{kl}(\mathscr{T})$ and $\bar{f}_{kl} = \max f_{kl}(\mathscr{T})$, where min and max are taken over all possible realizations of the activity durations. $f_{kl}(\mathscr{T})$ is the float of activity (k,l) in realization $\mathscr{T}$. Float $f_{kl}(\mathscr{T})$ is computed by means of formula the $f_{kl}(\mathscr{T}) = t^l_{kl}(\mathscr{T}) - t^e_{kl}(\mathscr{T}) - t_{kl}(\mathscr{T})$, where $t^e_{kl}(\mathscr{T})$ and $t^l_{kl}(\mathscr{T})$ are the earliest start and the latest finish times of activity $(k,l) \in A$ in realization $\mathscr{T}$ under the assumption of the minimum project duration in this realization. A float $f_{kl} \in F_{kl}$ if and only if there exists a realization of times $\mathscr{T}$ such that $f_{kl} = f_{kl}(\mathscr{T})$. The second problem considered in this paper is PAF: *compute bounds on float, $\underline{f}_{kl}$ and $\bar{f}_{kl}$, of a given activity $(k,l) \in A$.*

There is an obvious connection between PAPC and PAF:

Statement 2. *An activity $(k,l) \in A$ is possibly critical in S if and only if $\underline{f}_{kl} = 0$.*

3. Complexity results

In this section we explore the full picture of the computational complexity of the problems of evaluating the possible criticality and the related ones in a network with activity duration intervals.

The notions of the possible criticality were thoroughly investigated before [1,2]. It was shown there that the problems of determining an arbitrary possibly critical path, and of asserting whether a fixed path is possibly critical, can be solved in polynomial time ($\mathscr{O}(n^2)$ and $\mathscr{O}(n)$ for general and planar networks, respectively) and that PAPC for general networks is strongly NP-complete. Making use of the results obtained there and Statement 2, we can now state that PAF is strongly NP-hard for general networks. In case of series-parallel networks, PAPC and PAF were solved by Fargier et al. [3]. They provided $\mathcal{O}(n)$ algorithms for both problems. However, there is an important class of networks for which complexity results for the considered problems have not been obtained, namely, planar networks. The following results fill a gap in the computational complexity of PAPC and PAF. They give a complete answer to the question about the complexity of these problems in the case of planar networks.

Theorem 1. *The problem PAPC is* NP-*complete even if S is restricted to a planar graph.*

Proof. We begin by defining the problem PAPC for a planar network:

Input: A connected acyclic planar network $S(A, V)$, weights on the arcs (activity duration times) $(i, j) \in A$ are to be chosen from intervals $T_{ij} = [\underline{t}_{ij}, \bar{t}_{ij}]$, $\underline{t}_{ij} \geqslant 0$, a specified activity $(k, l) \in A$, two nodes 1 and n are distinguished as the initial and final node, respectively.

Question: Is the activity (k, l) possibly critical in S?

In [2], the authors have shown that PAPC for a general network belongs to NP. The same proof still goes for PAPC when S is restricted to be planar. It is based on Statement 1 and the fact that a path $p \in P$ is possibly critical in S if and only if it is critical in S in the usual sense when the duration intervals of all activities on p are at their upper bounds and the duration intervals of all the remaining activities are at their lower bounds.

We show now NP-hardness of PAPC for a planar network by reducing a certain modified PARTITION problem, denoted MPARTITION, to it.

The MPARTITION problem is:

Input: A finite set $\mathscr{A}$ of positive integers, $\mathscr{A} = \{a_1, \ldots, a_q\}$, having the overall sum of $2b$ and a positive integer $K < q$.

Question: Is there a subset $\mathscr{A}' \subset \mathscr{A}$ that sums up exactly to b and $|\mathscr{A}'| = K$?

It is well known that MPARTITION is NP-complete (see for instance [4] and comments on PARTITION given there).

We claim that an instance of MPARTITION is polynomially transformable to an instance of PAPC for a planar network.

The transformation proceeds as follows. To each instance of MPARTITION, we associate a network $S'(A', V')$ with $4q + 3$ nodes (events) labeled $1, 2, \ldots, 4q+3$ (see Fig. 1). Node $2i$, $i = 1, \ldots, q$, is adjacent to nodes $2i-1$, $2i+1$ and $2(2q-i+2)$. Arcs (activities) $(2i-1, 2i)$, $(2i, 2i+1)$ and $(2i, 2(2q-i+2))$, $i = 1, \ldots, q$, have weight intervals $[0, qa_i]$, $[0, 0]$ and $\left[q \sum_{j=i+1}^{q} a_j + q - i + 1,\ q \sum_{j=i+1}^{q} a_j + q - i + 1\right]$, respectively. The one-point intervals have been written in Fig. 1 as precise times. There are arcs $(2(2q - i+2) - 1, 2(2q-i+2))$ with weight interval $[0, 0]$ and $(2(2q - i + 2), 2(2q - i + 2) + 1)$ with weight interval $[0, qa_{q-i+1}]$, for $i = 1, \ldots, q$. Between nodes $2i - 1$ and $2i + 1$, and similarly, between $2(2q - i + 2) - 1$ and $2(2q - i + 2) + 1$ there are arcs $(2i - 1, 2i + 1)$ and $(2(2q-i+2)-1, 2(2q-i+2)+1)$ with weight interval $[1, 1]$, for $i = 1, \ldots, q$. Node $2q+2$ is adjacent to nodes 1, $2q + 1$, $2q + 3$ and $4q + 3$. The arcs $(2q + 1, 2q + 2)$ and $(2q + 2, 2q + 3)$ both have weight interval $[0, 0]$. The arcs $(1, 2q + 2)$ and $(2q + 2, 4q + 3)$ have weight intervals $[qb+q-K, qb+q-K]$ and $[qb+K, qb+K]$, respectively. There is an arc $(2q + 1, 2q + 3)$ having weight interval $[0, 0]$ and this arc is a specified activity. Nodes 1 and $4q + 3$ are the initial and final node. This completes the definition of $S'(A', V')$. Note that S' is a connected acyclic planar digraph. The construction of S' is done in a time bounded by polynomial in the size of MPARTITION.

Now we prove that there exists a subset $\mathscr{A}' \subset \mathscr{A}$ which sums up exactly to b and $|\mathscr{A}'| = K$ if and only if the activity $(2q+1, 2q+3)$ is possibly critical in S'.

$\Rightarrow$: Let $\mathscr{A}' \subset \mathscr{A}$ be a subset that sums up exactly to b and $|\mathscr{A}'| = K$. We show that there exists in S' a realization of times such that the specified activity $(2q + 1, 2q + 3)$ is critical, i.e. we determine a path $p \in P$ containing $(2q + 1, 2q + 3)$, which is critical in this realization.

Let us observe that each element $a_i \in \mathscr{A}$, $i = 1, \ldots, q$, corresponds to two triangles $(2i - 1, 2i, 2i + 1)$ and $(2(2q - i + 2) - 1, 2(2q - i + 2), 2(2q - i + 2) + 1)$ linked by arc $(2i, 2(2q-i+2))$ (see Fig. 1). If $a_i \in \mathscr{A}'$, then we include arcs $(2i - 1, 2i)$, $(2i, 2i + 1)$ (the right portion of the first triangle) and $(2(2q - i + 2) - 1, 2(2q - i + 2) + 1)$ (the left portion of the second triangle) in the path p. Otherwise ($a_i \in \mathscr{A} \setminus \mathscr{A}'$), we include arcs $(2i - 1, 2i + 1)$ (the left portion of the first triangle) and $(2(2q - i + 2) - 1, 2(2q - i + 2))$,

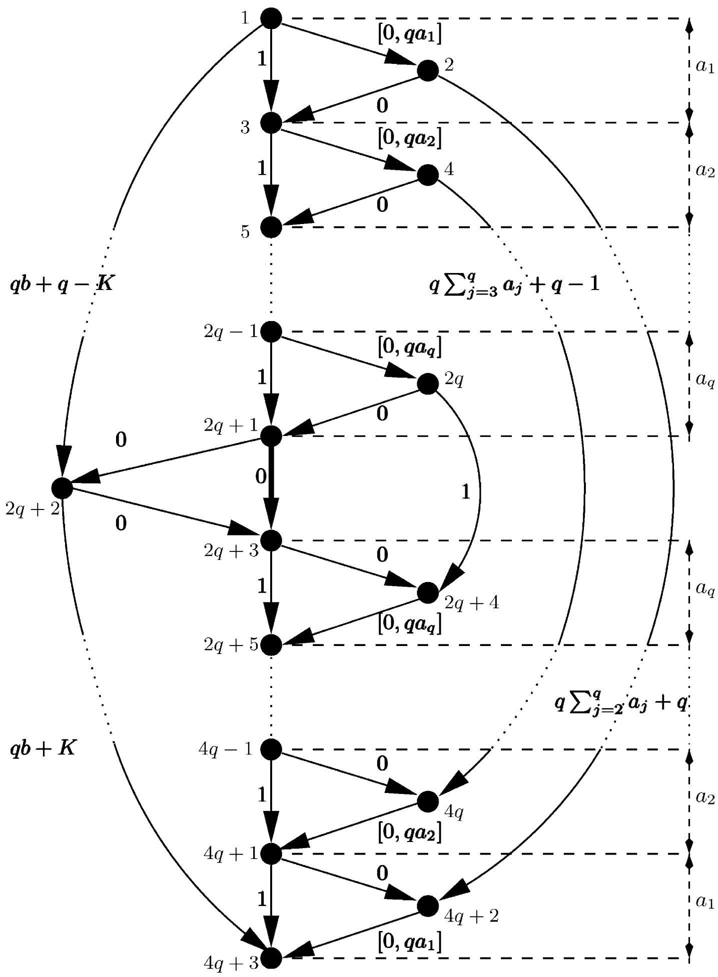

Fig. 1. The network S'.

$(2(2q-i+2), 2(2q-i+2)+1)$ (the right portion of the second triangle) in the path p. To complete p we add the specified arc $(2q+1, 2q+3)$. The construction of p is unique.

Let us consider realization $\mathscr{T}^*$ of the activity duration times in S' determined in the following way:

$$t_{ij}(\mathscr{T}^*) = \begin{cases} \bar{t}_{ij} & \text{if } (i,j) \in p, \\ \underline{t}_{ij} & \text{if } (i,j) \notin p. \end{cases} \quad (1)$$

We will show that path p is critical in S' in the realization of time $\mathscr{T}^*$ and thus arc $(2q+1, 2q+3)$ (that belongs to p) is a critical activity.

Note that each element a_i, $i=1,\dots,q$, belongs either to $\mathscr{A}'$ or to $\mathscr{A}\setminus\mathscr{A}'$. So, the determined path p must use either the right portion of the first triangle ($a_i \in \mathscr{A}$) and the left portion of the second one ($a_i \notin \mathscr{A}\setminus\mathscr{A}'$) or the left portion of the first ($a_i \notin \mathscr{A}$) and the right portion of the second triangle ($a_i \in \mathscr{A} \setminus \mathscr{A}'$). The subset $\mathscr{A}'$ sums up exactly to b and $|\mathscr{A}'| = K$. Then, the path p uses K times the right portions and $q-K$ times the left ones of the triangles, which precede arc $(2q+1, 2q+3)$. Hence, the length of the subpath of p leading from 1 to $2q+1$ is equal to $qb+q-K$ in $\mathscr{T}^*$. Similarly, as far as subset $\mathscr{A} \setminus \mathscr{A}'$ is concerned, which sums up exactly to b and $|\mathscr{A}\setminus\mathscr{A}'| = q-K$, the path p must use

$q-K$ times the right portions and K times the left ones of the triangles, which succeed arc $(2q+1, 2q+3)$. The length of the subpath of p leading from $2q+3$ to $4q+3$ is equal to $qb+K$ in the realization $\mathscr{T}^*$. Hence, the length of p is $2qb+q$. The rest of paths in S', i.e. paths that traverse parallel arcs to $(2q+1, 2q+3)$, have lengths at most $2qb+q$. One may enumerate these paths:

(1) paths containing one of arcs $(2i, 2(2q-i+2))$, for $i = 1, \dots, q$,
(2) paths containing arc $(2q+1, 2q+2)$,
(3) paths containing arc $(2q+2, 2q+3)$, and
(4) the path containing arcs $(1, 2q+2)$ and $(2q+2, 4q+3)$ (in this case the path is exactly of length $2qb+q$).

Consequently, the path p is one of the longest paths in S' under $\mathscr{T}^*$ and therefore it is critical in S' (it is possibly critical in S', see Definition 1). Statement 1 implies possible criticality of activity $(2q+1, 2q+3)$.

$\Longleftarrow$: Assume that the activity $(2q+1, 2q+3)$ is possibly critical in S'. Then, it is critical in the usual sense in S' under a certain realization of activity times, say $\mathscr{T}$ (see Definition 1). Hence, there exists a path p containing arc $(2q+1, 2q+3)$, which is critical in $\mathscr{T}$. Let us change the realization $\mathscr{T}$ by a new one $\mathscr{T}^*$, determined by formula (1). It is clear that for this new realization the path p is still critical in S'.

Now we give a property of the path p, which will be useful further in the proof.

Zigzag property: *The path p must use either arcs $(2i-1, 2i), (2i, 2i+1)$ (the right portion of the triangle, which precedes the specified arc) and arc $(2(2q-i+2)-1, 2(2q-i+2)+1)$ (the left portion of the triangle, which succeeds the specified arc) or arc $(2i-1, 2i+1)$ (the left portion) and arcs $(2(2q-i+2)-1, 2(2q-i+2)), (2(2q-i+2), 2(2q-i+2)+1)$ (the right portion), for $i = 1, \dots, q$.*

To prove Zigzag property, assume to the contrary that there exist i's, $1 \leqslant i \leqslant q$, such that path p traverses either arcs $(2i-1, 2i+1)$ and $(2(2q-i+2)-1, 2(2q-i+2)+1)$ simultaneously, or $(2i-1, 2i)$, $(2i, 2i+1)$, $(2(2q-i+2)-1, 2(2q-i+2))$ and $(2(2q-i+2), 2(2q-i+2)+1)$ simultaneously.

Consider the case when i such that path p uses arcs $(2i-1, 2i), (2i, 2i+1), (2(2q-i+2)-1, 2(2q-i+2))$ and $(2(2q-i+2), 2(2q-i+2)+1)$, simultaneously, does not exist. Then there exists at least one i such that path p must use arcs $(2i-1, 2i+1)$ and $(2(2q-i+2)-1, 2(2q-i+2)+1)$, simultaneously. We immediately arrive to a contradiction, since there exists path $(1, 2q+2), (2q+2, 4q+3)$ of length $2qb+q$ in the realization $\mathscr{T}^*$, which is longer than p.

Consider the case when there exists at least one i such that path p uses arcs $(2i-1, 2i)$, $(2i, 2i+1)$, $(2(2q-i+2)-1, 2(2q-i+2))$ and $(2(2q-i+2), 2(2q-i+2)+1)$, simultaneously. Let us choose the largest such i and denote it by i^*. Arc $(2i^*, 2(2q-i^*+2))$, parallel to the specified one, has weight $q\sum_{j=i^*+1}^{q} a_j + q - i^* + 1$. Since i^* is the largest i such that path p uses arcs $(2i-1, 2i)$, $(2i, 2i+1)$, $(2(2q-i+2)-1, 2(2q-i+2))$ and $(2(2q-i+2), 2(2q-i+2)+1)$, simultaneously, the length of the subpath of p, $(2i^*, 2i^*+1), \dots, (2q+1, 2q+3), \dots, (2(2q-i^*+2)-1, 2(2q-i^*+2))$, is at most $q\sum_{j=i^*+1}^{q} a_j + q - i^*$ in $\mathscr{T}^*$. This contradicts that p is critical, since there exists the path containing $(2i^*, 2(2q-i^*+2))$ with the length greater than the length of p.

Let us return to the main proof. Making use of Zigzag property we conclude that the path p must traverse q times the right and q times the left portion of the triangles in S'. So, for realization $\mathscr{T}^*$ p is of length $2qb+q$. We show that the subpath of p from node 1 to $2q+1$ has length l' equal to $qb+q-K$ in $\mathscr{T}^*$. In order to prove this, assume to the contrary that $l' < qb+q-K$. The subpath of path p from node $2q+3$ to $4q+3$ is of length $l'' > qb+K$. This implies the existence of a path with a length greater than $l'+l''$, i.e. a path containing arcs $(1, 2q+2), (2q+2, 2q+3)$ and the subpath of p from node $2q+3$ to $4q+3$. Similarly, if we assume that $l' > qb+q-K$, the subpath of path p from node $2q+3$ to $4q+3$ is of length $l'' < qb+K$. This implies the existence of a path longer than $l'+l''$. This path contains the subpath of p from node 1 to $2q+1$ and arcs $(2q+1, 2q+2), (2q+2, 4q+3)$.

The proof that the subpath of path p from node $2q+3$ to $4q+3$ is of length $qb+K$ in $\mathscr{T}^*$ is similar.

Let us determine a subset $\mathscr{A}'$. If the subpath of p from node for 1 to $2q+1$ uses arc $(2i-1, 2i)$ (the right portion of the triangle) then a_i belongs to $\mathscr{A}'$, $i = 1, \dots, q$. Otherwise (it uses $(2i-1, 2i+1)$) a_i belongs $\mathscr{A} \setminus \mathscr{A}'$. Zigzag property guarantees that a_i does not belong to $\mathscr{A}'$ and $\mathscr{A} \setminus \mathscr{A}'$ simultaneously. Since the subpath is of length $qb+q-K$ in $\mathscr{T}^*$ and $qa_i > q-K$, for $i = 1, \dots, q$, it must use $q-K$ times

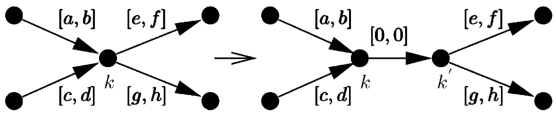

Fig. 2. The transformation in the proof of Theorem 2.

the left portions and K times the right portion of the triangles. This means that K elements belong to $\mathscr{A}'$ ($|\mathscr{A}'| = K$). Moreover, $\mathscr{A}'$ sums up exactly to b. Note that $q-K$ arcs from the subpath have weight intervals $[1,1]$ (the left portion of the triangles) and they sum up to $q-K$. The rest of the arcs from K right portions of the triangles, corresponding to the elements that belong to $\mathscr{A}'$, have a sum of weights equal to qb. Hence, $\sum_{\{a_i|a_i\in\mathscr{A}',1\leqslant i\leqslant q\}} a_i = b$. This completes the proof. □

The following theorem shows that planarity and bounded node degree (the degree of a node is the sum of the number of its incoming and outgoing arcs) are not sufficient to keep PAPC from being NP-complete.

Theorem 2. *The problem PAPC is* NP-*complete even if S is restricted to a planar graph with node degree three.*

Proof. The proof is analogous to that of Theorem 1. It is evident that PAPC for a planar network with node degree three belongs to NP. We need to show that an instance of MPARTITION is polynomially transformable to an instance of PAPC for a planar network with node degree three. The transformation is divided into two parts. In the first one we associate to each instance of MPARTITION a directed, acyclic, planar network $S'(A', V')$. The construction of S' is the same as in the proof of Theorem 1. In the second part we transform network S', which has maximum node degree four, into a planar network $S''(A'', V'')$ with node degree three. To obtain S'', it is enough to split each node $k \in V'$ with degree four in S' (see Fig. 1) by inserting an arc (k, k') having weight interval $[0,0]$ (see Fig. 2). It is easily seen that S'' is still a connected acyclic planar digraph and each node has degree three. The construction of S'' is done in a time bounded by polynomial in the size of MPARTITION.

It remains to show that there exists a subset $\mathscr{A}' \subset \mathscr{A}$ that sums up exactly to b and $|\mathscr{A}'| = K$ if and only if the activity $(2q+1', 2q+3)$ is possibly critical in S''. The proof of this equivalence proceeds in the same manner as for the network S' constructed in the proof of Theorem 1. Thus, PAPC is NP-complete for a planar network with node degree three. □

From Theorem 2 and Statement 2 we immediately obtain the computational complexity of PAF.

Corollary 1. *The problem PAF is* NP-*hard even if S is a planar graph with node degree three.*

4. Conclusion

We have shown that the problems of evaluating the possible criticality of an activity and of computing bounds on the float of an activity in networks with duration time intervals remain NP-hard (NP-complete for the possible criticality) even when networks are planar and have a bounded node degree. Thus, these problems are unlikely to have efficient algorithms.

It is worth pointing out that all results presented here may be of substantial use in problems of activity criticality in networks with activity duration times given in the form of fuzzy intervals, when they are stated in the framework of *possibility theory* (see [1,3]). Namely, the problems of computing the possibility degree that an activity is critical, and of determining the fuzzy float of an activity, also turn out to be NP-hard in the case of planar networks. This follows from the fact that every fuzzy interval number can be decomposed into a family of intervals according to its level-cuts. Hence, all the interval problems analyzed in the paper are particular cases of the corresponding fuzzy ones.

Acknowledgements

This work was supported by Grant No. 7T11F02120 from the State Committee for Scientific Research (Komitet Badań Naukowych).

References

[1] S. Chanas, P. Zieliński, Critical path analysis in the network with fuzzy activity times, Fuzzy Sets and Systems 122 (2001) 195–204.

[2] S. Chanas, P. Zieliński, The computational complexity of the criticality problems in a network with interval activity times, European J. Oper. Res. 136 (3) (2002) 541–550.
[3] H. Fargier, V. Galvagnon, D. Dubois, Fuzzy PERT in series-parallel graphs, Ninth IEEE International Conference on Fuzzy Systems, San Antonio, TX, 2000, pp. 717–722.
[4] M.R. Garey, D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, Freeman, San Francisco, CA, 1979.
[5] J.E. Kelley, M.R. Walker, Critical path planning and scheduling, in: Proceedings of the Eastern Joint Computational Conference, Boston, MA, Vol. 16, 1959, pp. 160–172.
[6] H. Yaman, O.E. Karasan, M.C. Pinar, The robust spanning tree problem with interval data, Oper. Res. Lett. 29 (2001) 31–40.