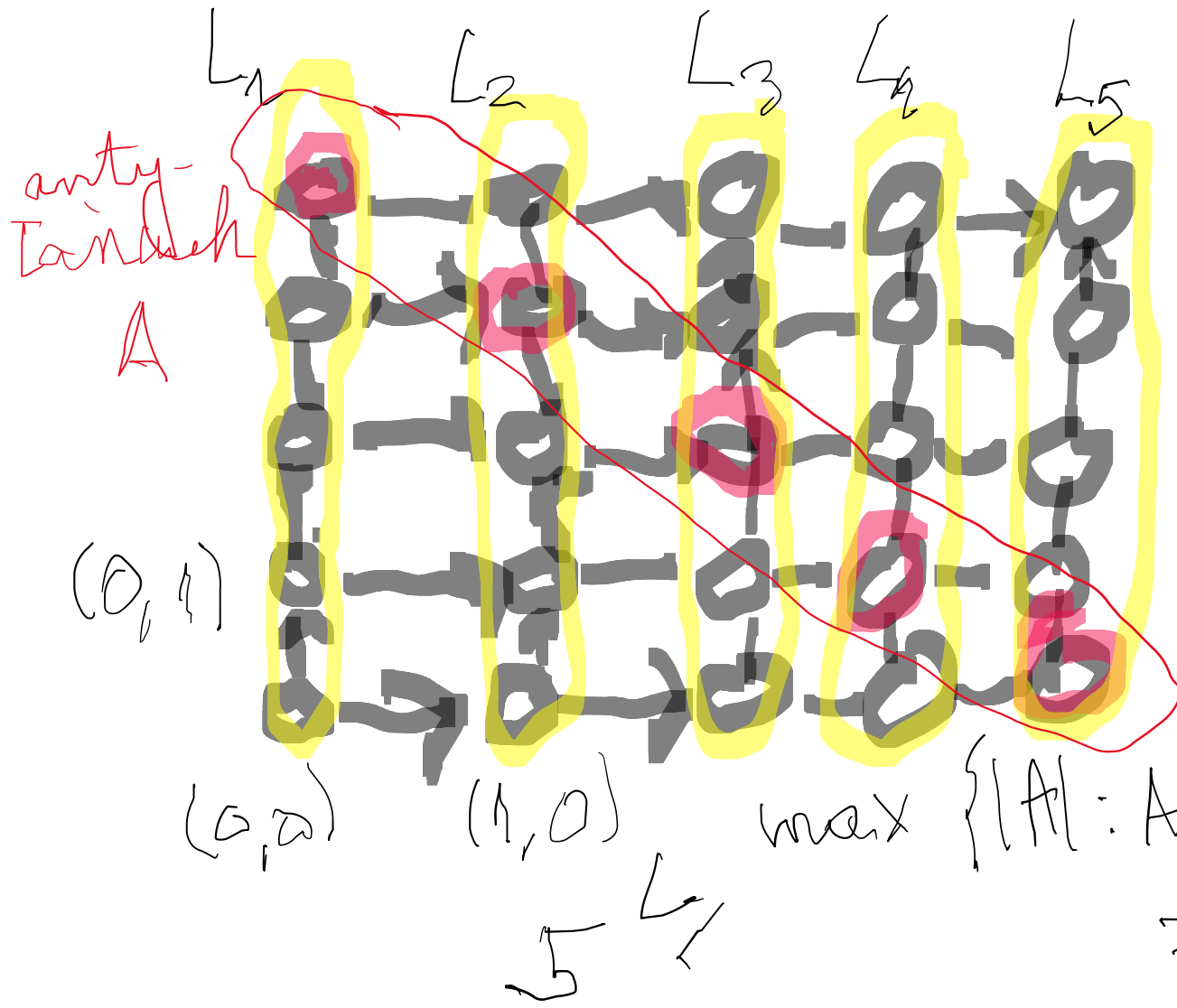


$$\Omega = \{0, \dots, n\}^2$$

$$(a, b) \prec (c, d) \iff (a \leq c) \wedge (b \leq d)$$



$$(0,4) \quad |A| = 5$$

$$|\mathcal{L}| = |\{L_1, \dots, L_5\}| = 5$$

Dilworth's thm

$$\max \{|A| : A \text{-antichain}\} = \min \{|\mathcal{L}| : \mathcal{L} \dots\}$$

$$f: X \rightarrow \mathbb{R}$$

$$g: Y \rightarrow \mathbb{R}$$

$$\underbrace{\alpha = \max_{x \in X} \{f(x)\}}_{\text{max}} = \min_{y \in Y} \{g(y)\}$$

$$a \in X, b \in Y:$$

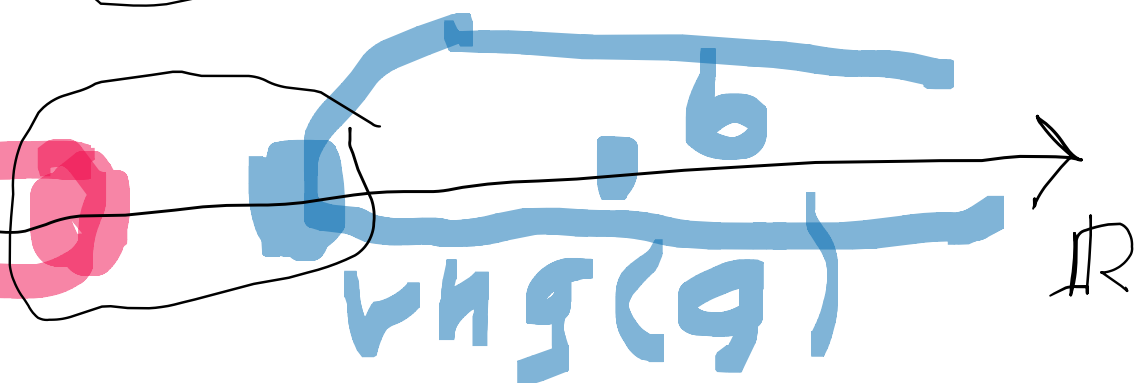
$$f(a) = g(b)$$

$$g(b) \geq \min \{g(y) \in Y\} = \alpha$$

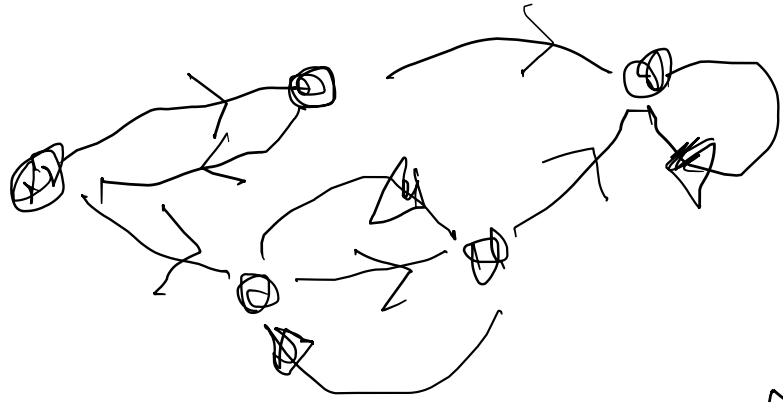
$$\alpha \leq f(a) = g(b) \geq \alpha$$

for all $x \in X$
 $f(x) \leq g(y)$

$$f(a) \leq \alpha$$



Grafy skierowane



$$(V, E, \varphi)$$

fgr
Lacydenys

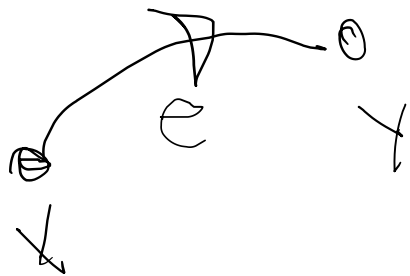
$$\varphi: E \rightarrow V \times V$$

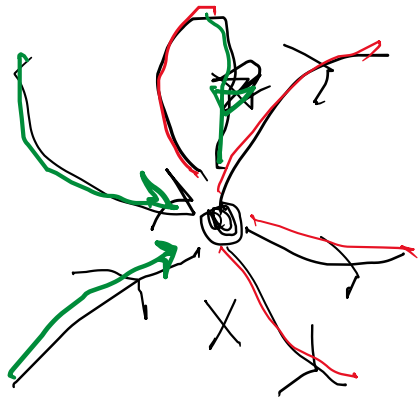
$$\varphi(e) = (x, y)$$

$$x \leftarrow \text{ogon } e$$
$$y \leftarrow \text{glowa } e$$

$$\text{fst}(e) = x \quad \text{snd}(e) = y$$

(2 Haskell'a)





$$\text{deg}^+(x) = |\{e : \text{fst}(e) = x\}|$$

$$\text{deg}^+(x) = 4$$

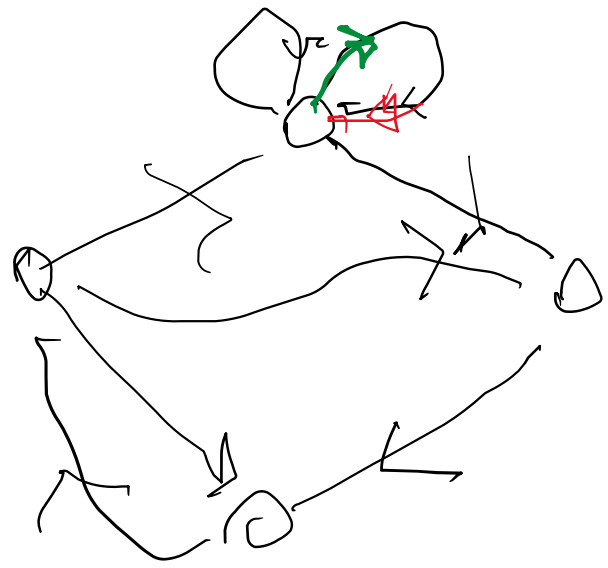
$$\text{deg}^-(x) = |\{e \in E : \text{snd}(e) = x\}|$$

$$\text{deg}^-(x) = 3$$

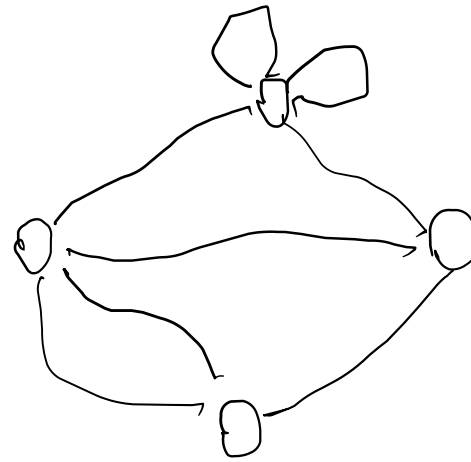
$$E = \bigcup_{x \in V} \{e \in E : \text{fst}(e) = x\}$$

$$|E| = \sum_{x \in V} |\{e : \text{fst}(e) = x\}| = \sum_{x \in V} \text{deg}^+(x)$$

$$|E| = \sum_{x \in V} \text{deg}^-(x) \quad \text{TW.}$$



digraf G

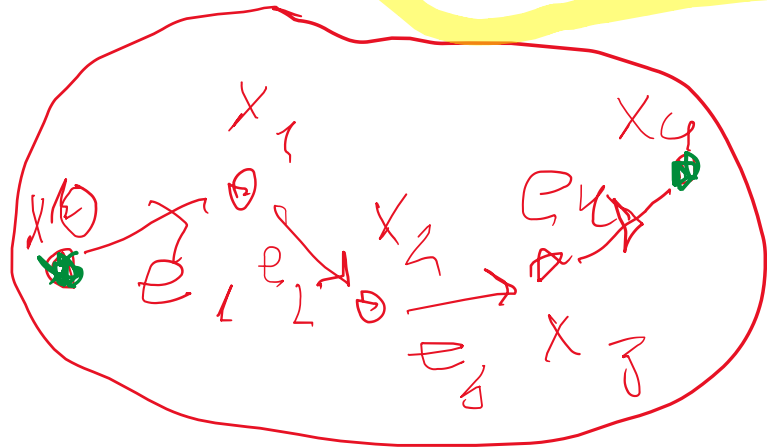


szkieleł G

209

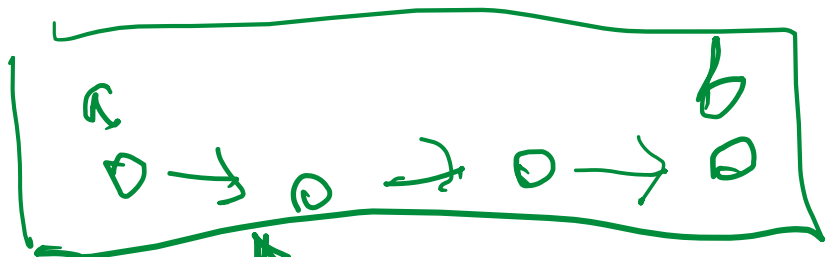
$$\text{deg}_G(x) = \text{deg}_G^+(x) + \text{deg}_G^-(x)$$

ścieżka



$\mathcal{P}: x_1 e_1 x_2 e_2 x_3 e_3 x_4 e_4 x_4$

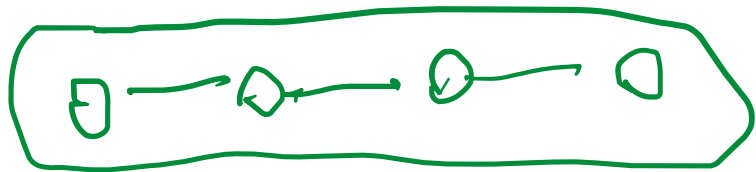
$$dt(\mathcal{P}) = 4$$



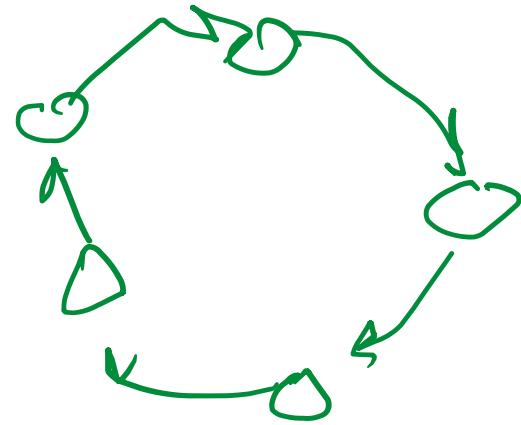
\hookrightarrow jest silniejsza od a
 \parallel do b

$a \gg b$

$\neg (b \gg a)$



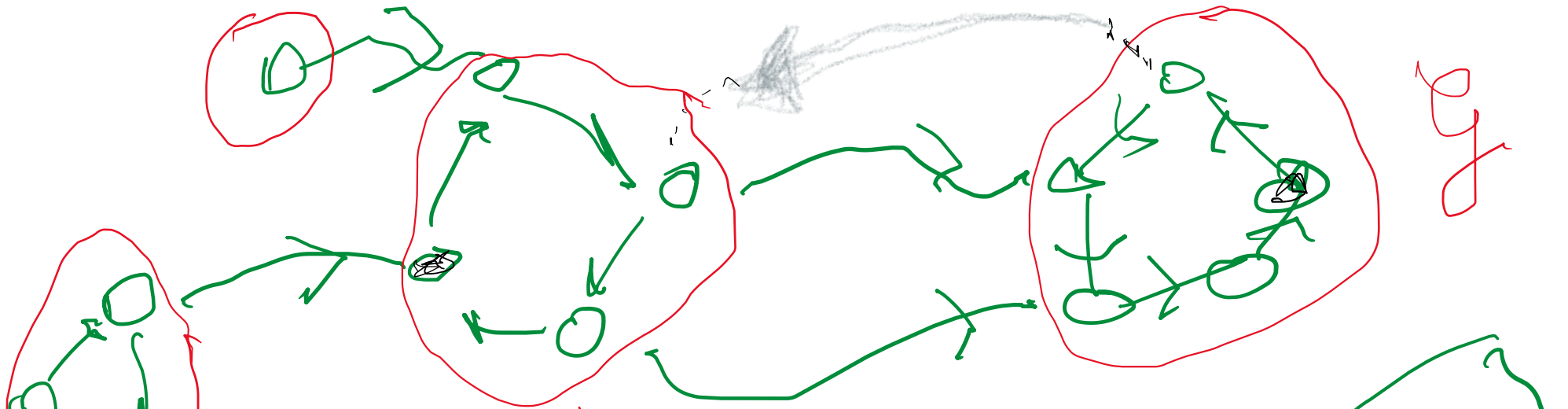
\hookrightarrow spojone



\hookrightarrow

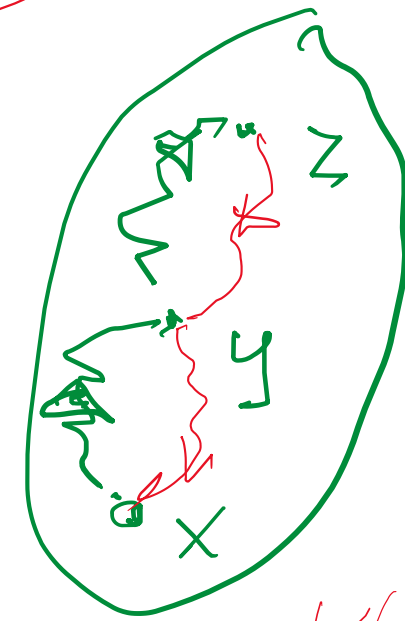
g-silnie spojony

$\forall x, y \in V : x \gg y$



of

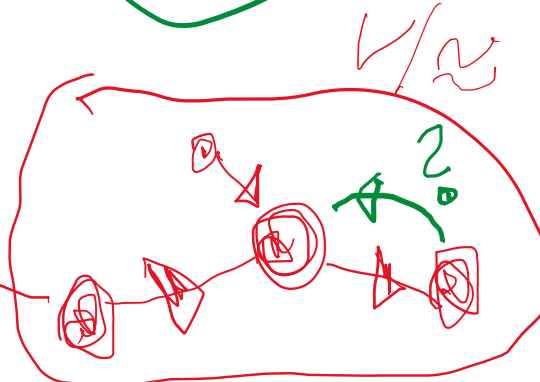
silnia spogne korp
(skladow)



$$x \approx y \equiv (x \gg y) \wedge (y \ll x)$$

met.
wzajem.

- $x \approx x$
- $x \approx y \rightarrow y \approx x$
- $x \approx y \wedge y \approx z \rightarrow x \approx z$

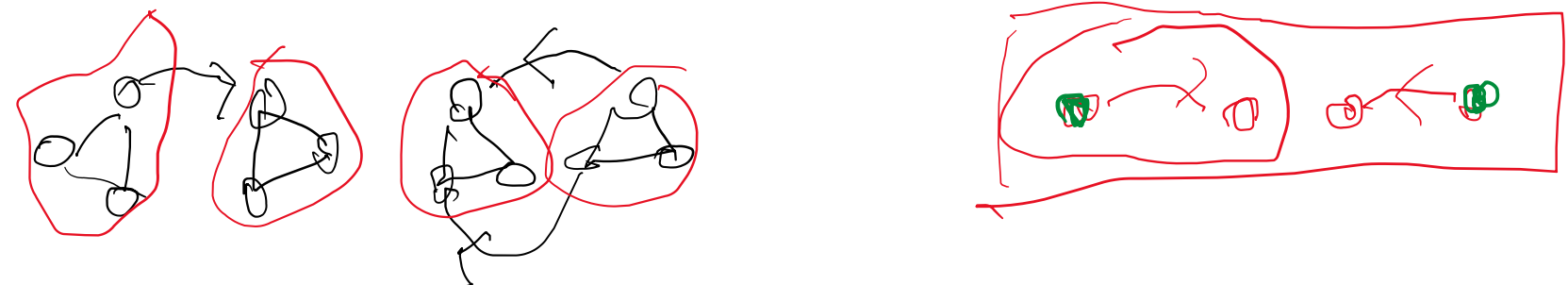


kondens. cy

dag \equiv directed acyclic graphs

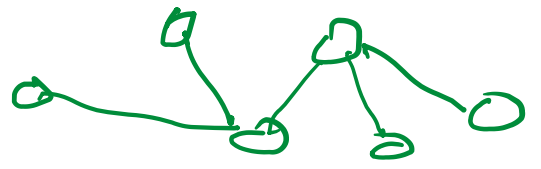
FAKT

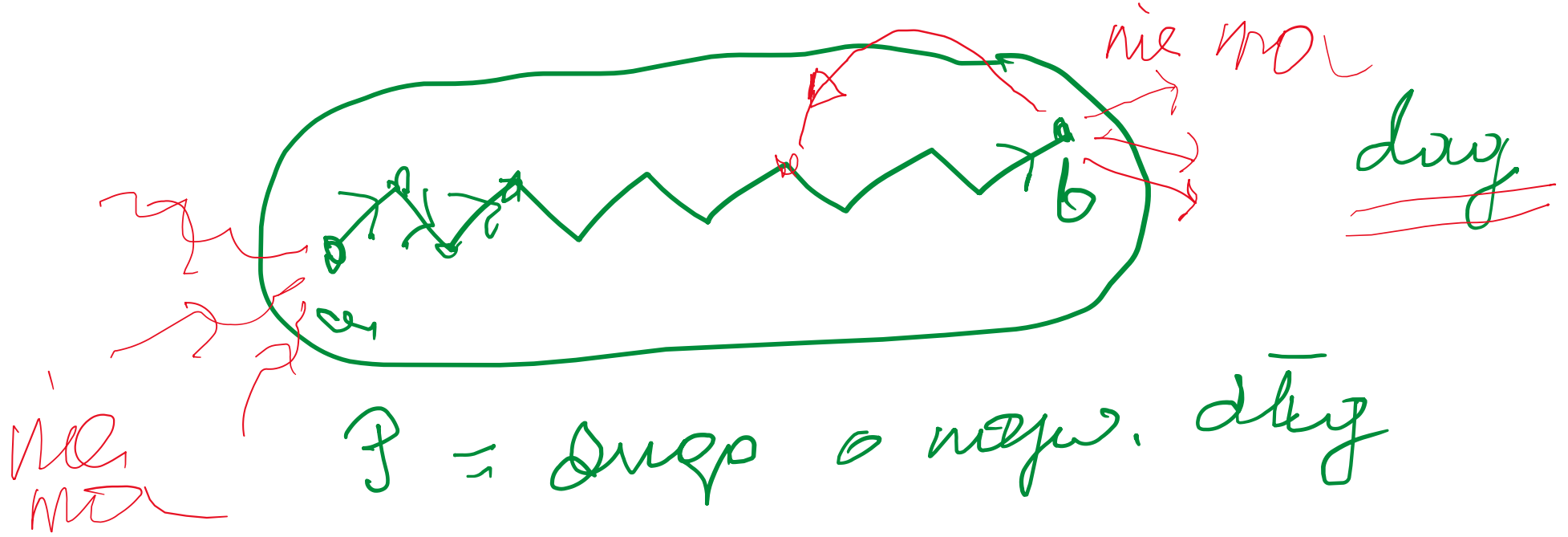
$\forall (G) \exists$ topological order



\exists topo order \iff $\forall x, \text{in } \text{dep}^-(x) = \emptyset$
 $\text{topo order} \iff \forall x, \text{in } \text{dep}^+(x) = \emptyset$

FAKT: G - dag $\rightarrow \exists$ topo order
 \exists topological order

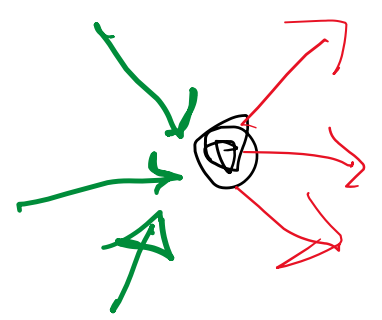


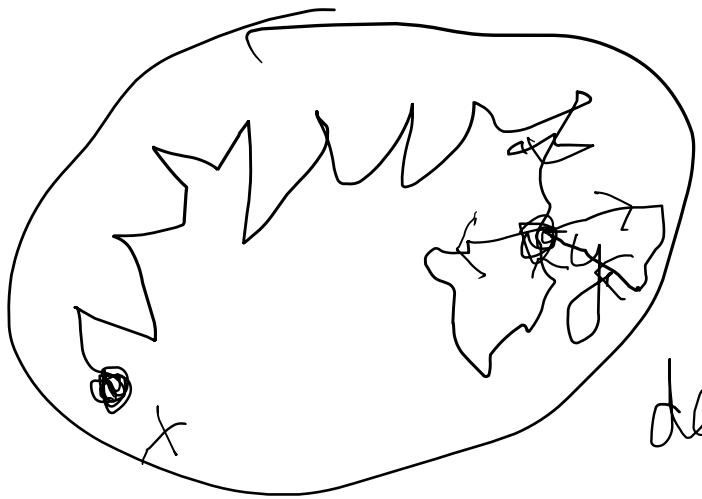


Ⓟ $\text{długość jest eulerska}$

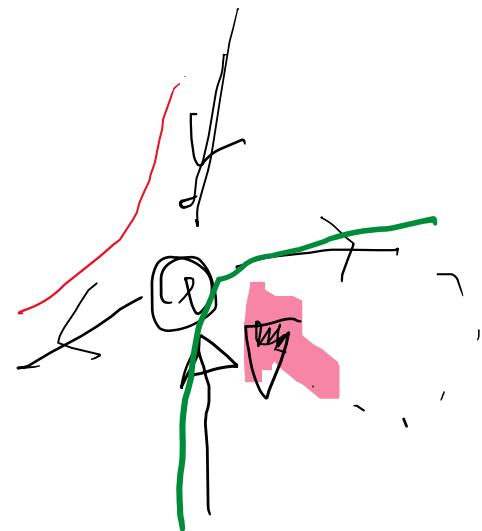
$\forall x \quad \text{deg}^{-}(x) = \text{deg}^{+}(x) + \text{spójny}$

zadanie!!!





$$\text{deg}^-(a) = \text{deg}^+(a) + 1$$



$$\text{deg}^-(x) + 1 = \text{deg}^+(a)$$

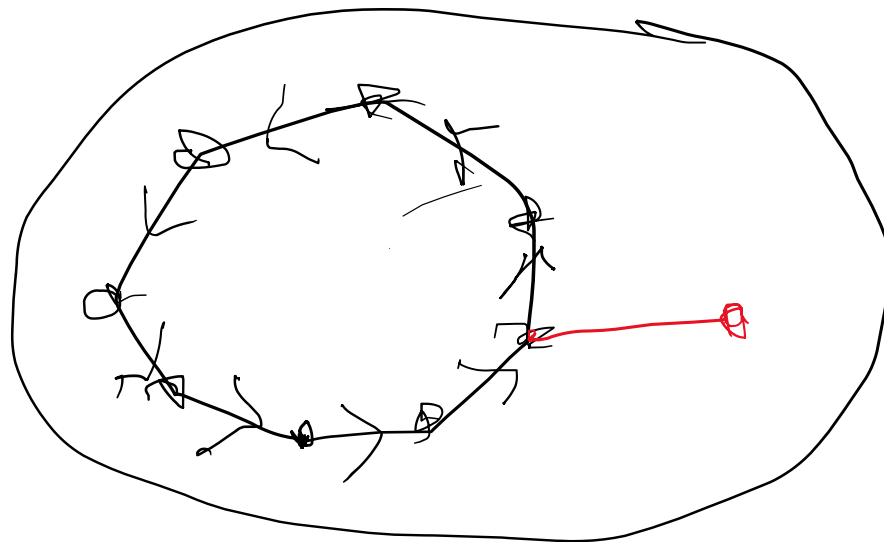
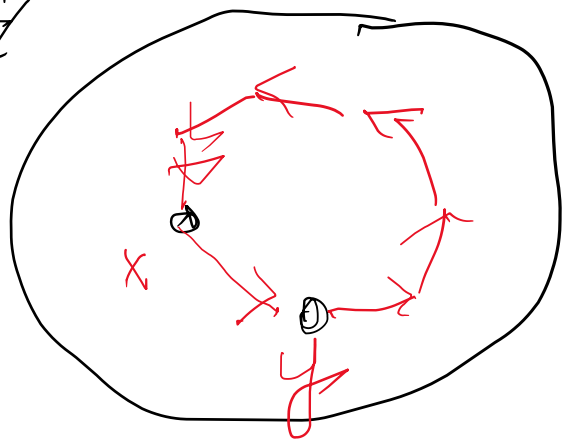
G - graf : G jest orientowany

istnieje orient G dający
 graf suma spójny

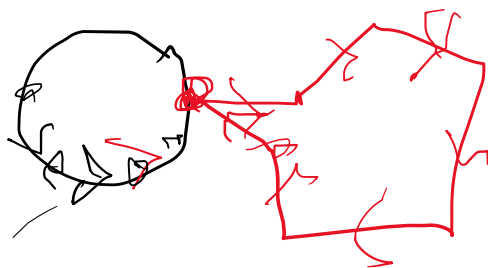
\mathcal{G} -orient

$\equiv \mathcal{G}$ -strong 1

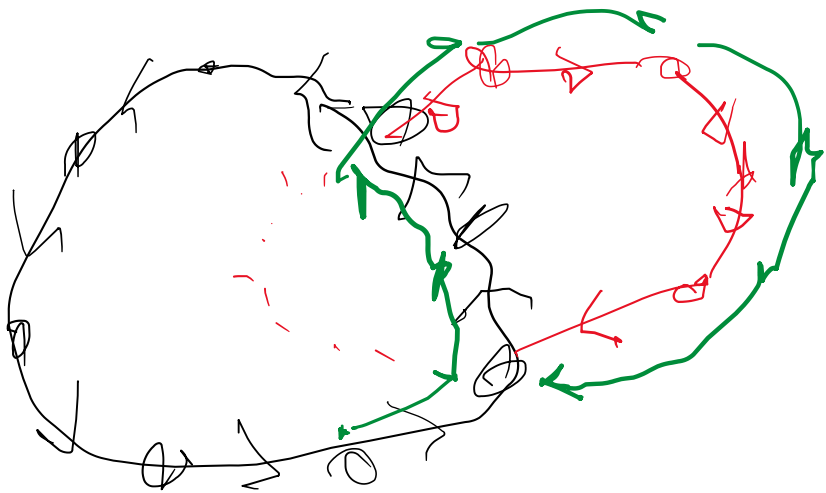
ward level \mathcal{D}
just no cycle



case 1



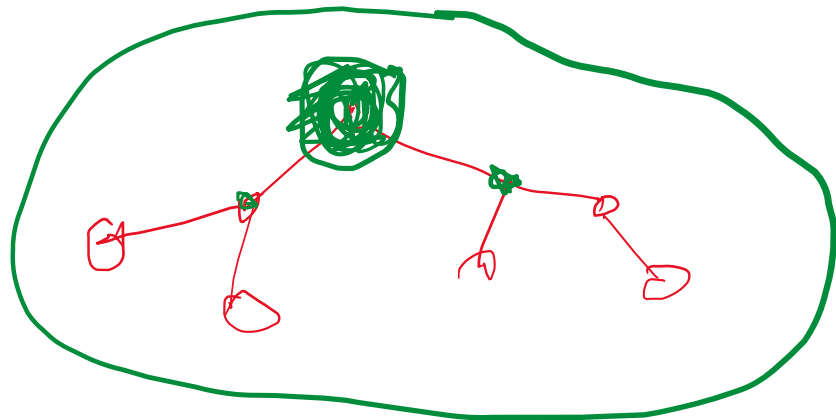
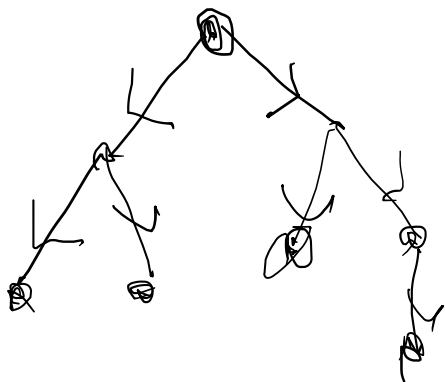
Case 2.

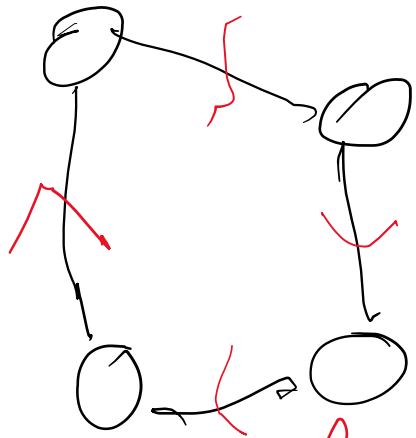


Area : graf : spojny + acyklicny

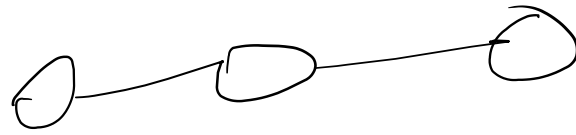


C2, not :





orientacja



2 mierzalności

TW. G jest orient. $\iff G$ jest spójny

\wedge każde krawędź jest

me cyklu

$\iff G$ -spójny $\wedge G$ nie ma most