Matrix multiplication

$$\boxed{A} \circ \boxed{B} \Rightarrow \boxed{C=A\cdot B}$$

$$c_{ik} = \sum_{j=1}^{n} a_{i\underline{j}}\cdot b_{\underline{j}k}$$

$$A = [a_{ij}]_{i,j=1..n} \qquad (A, i, j, a_{ij})$$

$$B = [b_{jk}]_{j,k=1..n} \qquad (B, j, k, b_{jk})$$

$$\left\{ \begin{array}{l} a_{ij} \neq 0 \\ b_{jk} \neq 0 \end{array} \right.$$

M1 map:

$$(A, i, \underline{j}, a_{ij}) \longrightarrow (j, (A, i, a_{ij}))$$

$$(B, j, k, b_{jk}) \longrightarrow (j, (B, k, b_{jk}))$$

reduce (j, L)

$$a_{1j}\cdot b_{j1}$$

$$L = [\underline{(A, 1, a_{1j})}, (A, 2, a_{2j}) \ldots \mid \underline{(B, 1, b_{j1})}, (B, 2, b_{j2}), \ldots]$$

$$j:\ L = [\underbrace{(A, 1, a_{1j}), \ldots}_{L_1} \mid \underbrace{(B, 1, b_{j1}), \ldots}_{L_2}]$$

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reduce(j, L) {
    for i, k = 1..n do
        emit((i,k), $a_{ij} \cdot b_{jk}$));
}
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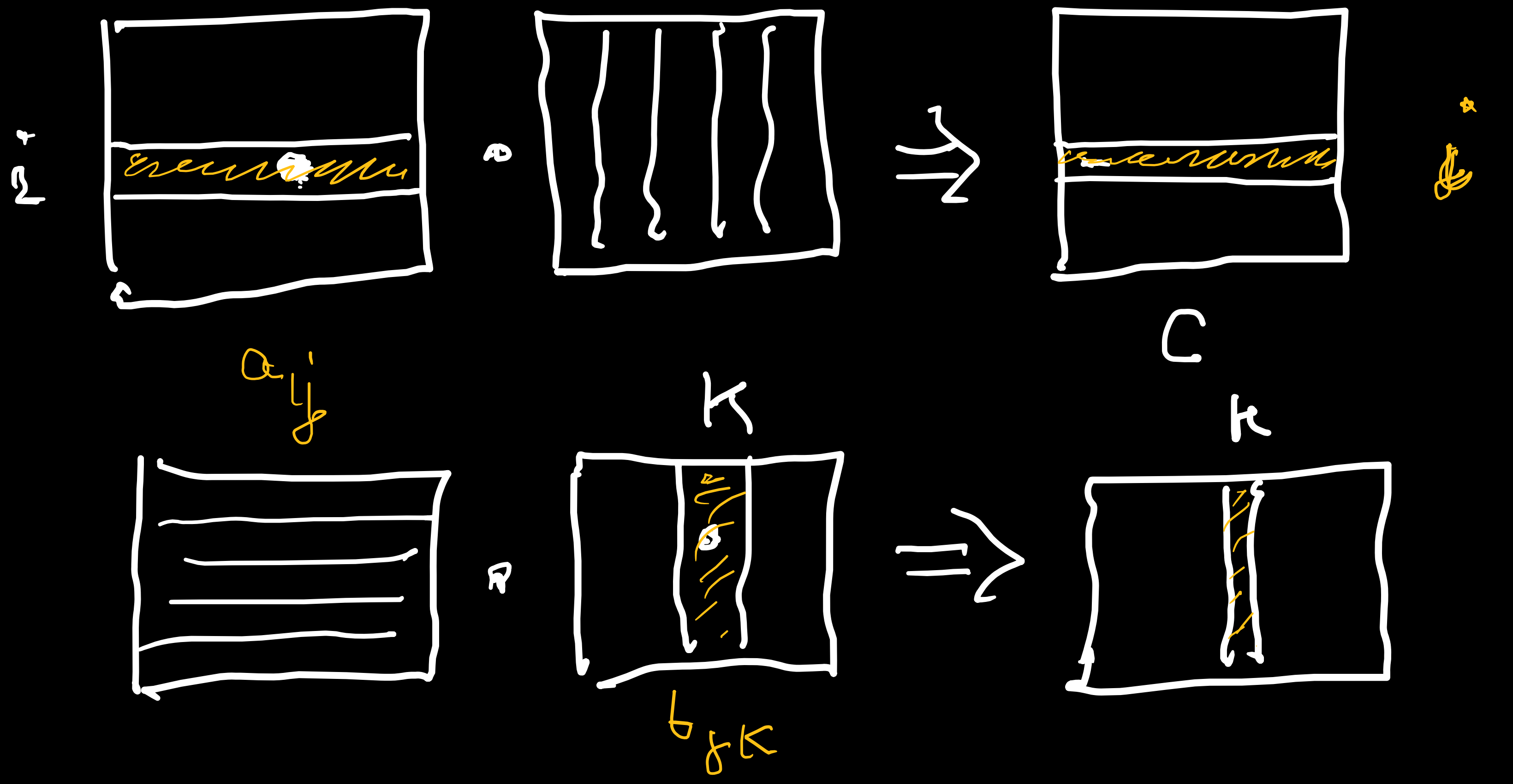
$$\left] \begin{cases} [((1,1), a_{11}\, b_{11}) \\ [((1,1), a_{12}\, b_{21}) \\ (((1,1), a_{13}\, b_{31}) \ldots \end{cases} \right.$$

$$L = [(A, 1, a_{1j}], (A, 3, a_{3j}), (A, 10, a_{10j}) \ldots]$$

USE M-R second time to fuse data

M2



$$\text{map}: \begin{cases} (A, i, j, a_{ij}) \longrightarrow \text{forall } k=1..n \text{ do emit}((i,k),(A,j,a_{ij})) \\ (B, j, k, b_{jk}) \longrightarrow \text{forall } i=1..n \text{ do emit}((i,k),(B,j,b_{jk})) \end{cases}$$

$$\text{reducer}((i,k), L)\ \{\ L = \underset{A}{L1} \,|\, \underset{B}{L2}\ ;\ \text{emit}\left((i,k), \sum_{j} a_{ij} a_{jk}\right);\ \}$$

CLASSICAL SOLUTION

A, B almost fits in memory

$n = 10^5$ $n^2 = 10^{10} \cong 10$ GB

$(M_{n \times n}(\mathbb{R}), +, \cdot, 0, 1) \leftarrow$ ring

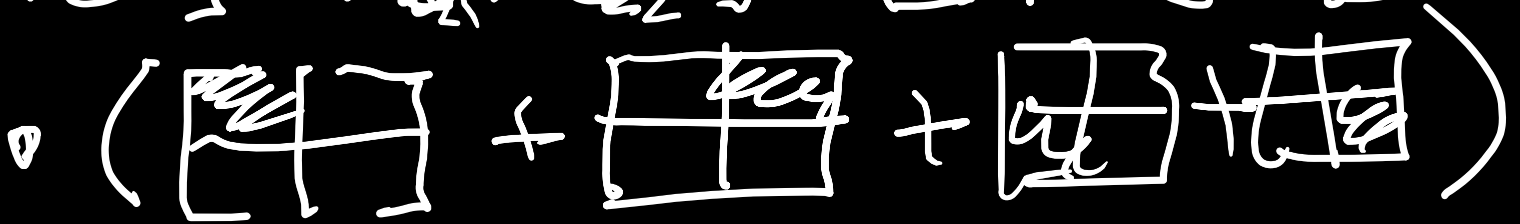
6 - bytes repres. of numbers

TOTALLY : 60 GB for A, B

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} =$$

$n = 2m$

$$\left(\begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix} \right)$$



$$\begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} B_{11} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} A_{11} \cdot B_{11} & 0 \\ 0 & 0 \end{bmatrix}$$

EX.

$$\begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix} \cdot \begin{bmatrix} 0 & B_{12} \\ 0 & 0 \end{bmatrix} = ?$$