

Mnożenie macierzy

$$\boxed{A} \cdot \boxed{B} = \boxed{C}$$

$$A = [a_{ij}] \quad B = [b_{jk}]$$

W1 (złączenie)

$$\left\{ \begin{array}{l} \text{mapper: } [A, i, j, a_{ij}] \rightarrow (j, (A, i, a_{ij})) \\ \quad \quad \quad (B, j, k, b_{jk}) \rightarrow (j, (B, k, b_{jk})) \end{array} \right.$$

$$\text{reduce}(j, L)$$

$$L = [\overset{L_1}{(A, 1, a_{1j}), (A, 2, a_{2j}) \dots } \overset{L_2}{(B, 1, b_{j1}), (B, 2, b_{j2}) \dots }]$$

$$c_{jk} = \sum_{i=1}^n a_{ij} \cdot b_{jk}$$

$$\text{data: } \begin{cases} (A, i, j, a_{ij}) & a_{ij} \neq 0 \\ (B, j, k, b_{jk}) & b_{jk} \neq 0 \end{cases}$$

for $l=1$ TO n
for $k=1$ TO n
 $\text{emit}((l, k), a_{ij} \cdot b_{jk})$
ZASTOSUJ II WZ
GROUP REDUCE



maps

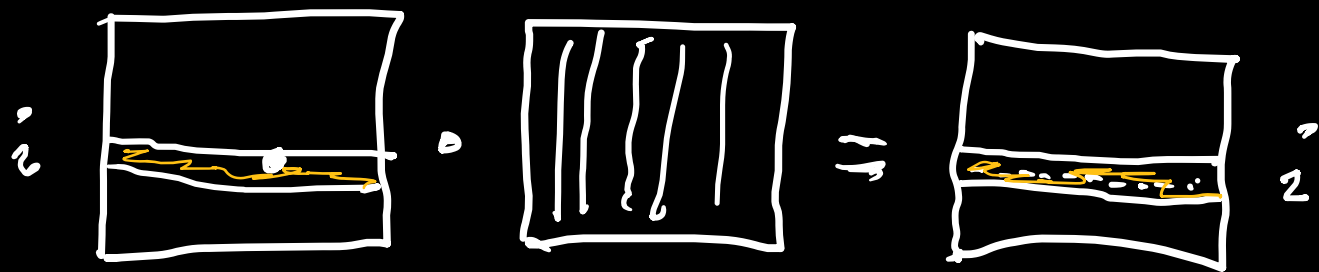
\uparrow
 with labels
 $O(m) + O(n) = O(n)$

$(L, K) \rightsquigarrow \underbrace{a_{L1} \cdot b_{1K}, a_{L2} \cdot b_{2K}}_{O(n)}$

$(A_{11}, \dots, a_{1j}) \rightarrow$

$(j, (A_{11}, \dots, a_{1j}))$

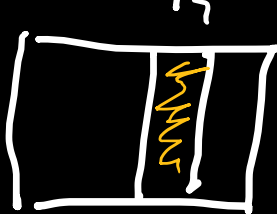
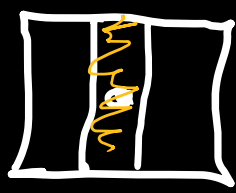
W2



A

B

C



$$c_{ik} = \sum_j a_{ij} \cdot b_{jk}$$

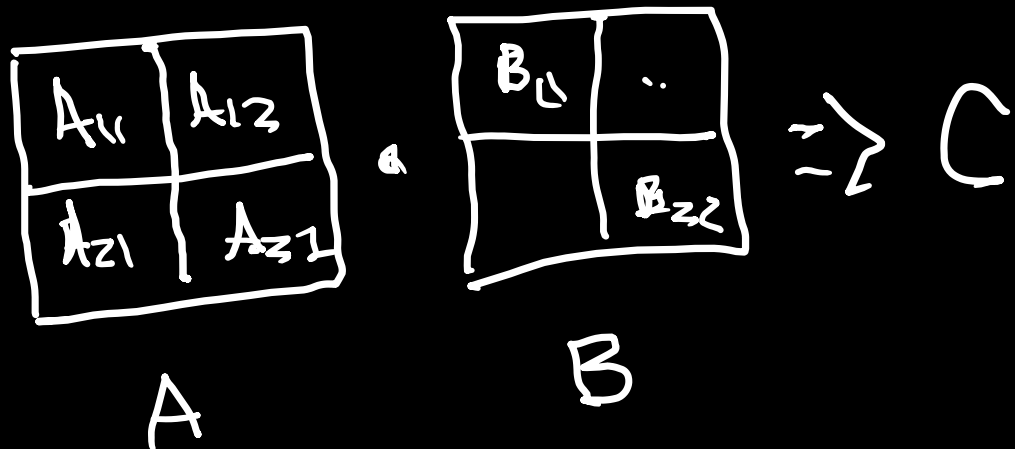
$$\begin{bmatrix} c_{1k} \\ c_{2k} \\ \vdots \\ c_{lk} \end{bmatrix} \cdot [k_1 \dots k_n] = [c_{ik}]$$

map $\left\{ \begin{array}{l} (A, L, j, a_{ij}) \rightarrow \text{for } k=1 \text{ to } n \text{ do} \\ \quad \text{emit } (i, k), (A, j, a_{ij}) \\ (B, j, k, b_{jk}) \rightarrow \text{for } w=1 \text{ to } L \text{ do} \\ \quad \text{emit } (w, k), (B, j, b_{jk}) \end{array} \right.$

reduce $((i, k), L)$ w L mapy wszystkie info do wyliczenia c_{ik}

KLASYCZNE POTĘŻO.

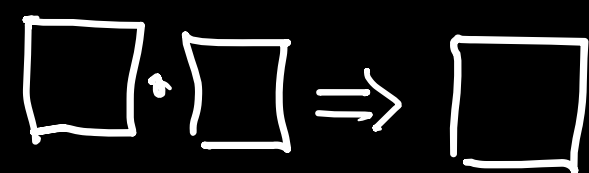
$$n = 2 \cdot m$$



$$m \approx 10^5$$

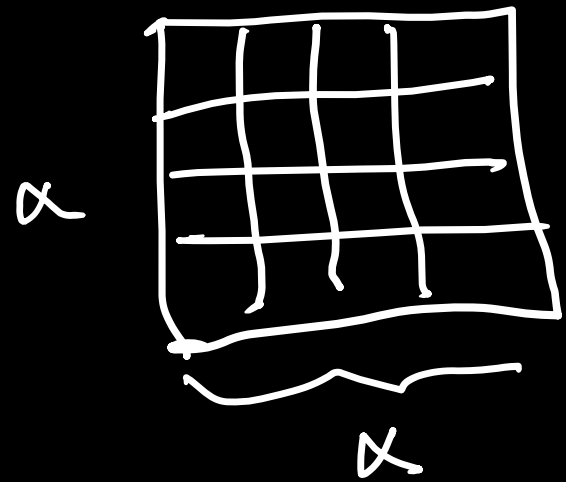
$$m^2 \approx 10^{10}$$

$$10^{10} \cdot 6 = 10^9 \cdot 60 \approx 60GB$$



$$A \cdot B = \left(\begin{array}{c|c} A_{11} & 0 \\ \hline 0 & 0 \end{array} + \begin{array}{c|c} 0 & A_{21} \\ \hline 0 & 0 \end{array} + \begin{array}{c|c} \text{shaded} & \text{shaded} \\ \hline \text{shaded} & \text{shaded} \end{array} + \begin{array}{c|c} \text{shaded} & \text{shaded} \\ \hline \text{shaded} & \text{shaded} \end{array} \right) \cdot \left(\begin{array}{c|c} B_{11} & 0 \\ \hline 0 & 0 \end{array} + \dots + \begin{array}{c|c} \text{shaded} & \text{shaded} \\ \hline \text{shaded} & \text{shaded} \end{array} \right)$$

$$\left. \begin{array}{c} \begin{array}{c|c} \text{shaded} & \text{shaded} \\ \hline \text{shaded} & \text{shaded} \end{array} \cdot \begin{array}{c|c} \text{shaded} & \text{shaded} \\ \hline \text{shaded} & \text{shaded} \end{array} \\ \text{A}_{11} \cdot \text{B}_{12} \end{array} \right\} = \left\{ \begin{array}{l} \text{A}_{11} \cdot \text{B}_{11} + \text{A}_{12} \cdot \text{B}_{21} \quad | \quad \text{A}_{11} \cdot \text{B}_{12} + \text{A}_{12} \cdot \text{B}_{22} \\ \text{A}_{21} \cdot \text{B}_{11} + \text{A}_{22} \cdot \text{B}_{21} \quad | \quad \text{A}_{21} \cdot \text{B}_{12} + \text{A}_{22} \cdot \text{B}_{22} \end{array} \right.$$



$$A \cdot B$$

$n \times n$



$$A_{ij} \cdot B_{jk}$$

$$\left(\frac{n}{\alpha}\right)^2 = \frac{n^2}{\alpha^2}$$

$\mathbb{R} = (\mathbb{R}, +, \cdot, 0, 1)$ ← pierścien $\alpha 1$

$M_{n \times n}(\mathbb{R})$ = zbiór macierzy wymiaru $n \times n$ z el. z \mathbb{R}

↑
pierścien

w pamięci
używamy dobrego alg. kerolca
 $O(n^{2.8})$ ← strassen



$$\cong_{\mathbb{Z}O} M_{k \cdot n \times k \cdot n}(\mathbb{R})$$



map-reduce : x_1, \dots, x_n $n \sim$ wielkie

- funkcje wazujące z rodzaju uniwers.

$$\sum_{i=1}^n x_i$$

{wynik jest niepowtarzalny}

- błędy numeryczne

$$\bar{x} = [\underbrace{10^{12}, 10^{13}, \dots}_{}, 10^{-9}, 10^{-12}, 10^{-11}, 10^{12}, 10^{13}, \dots]$$

\bar{y} - sortuj \bar{x} od najm. do najw.
oblicz \bar{y} .

$$\text{map}(x) \{ \text{emit}(\text{ceil}(\log_2(x)), x) \}$$

reducer : (k, L) \leftarrow wartość x

(P) Top 10

DANE: x_1, x_2, \dots, x_n
l. liczb

```
mapper, unit(L) {  
    Z = [ -∞, -∞, ..., -∞ ]  
}
```

```
mapper, ou Get (x: double) {
```

1. $m \leftarrow \min(Z)$

2. $(m < x) \rightarrow \{ \text{usunij } m \text{ z } Z; \text{ zastap } m \text{ przez } x \}$

```
}
```

```
mapper, finalize() {
```

emit(1, Z)

CEL: wyznaczc 10 najw. liczb.

```
reducer (1, L) {
```

1. scal L

2. sort

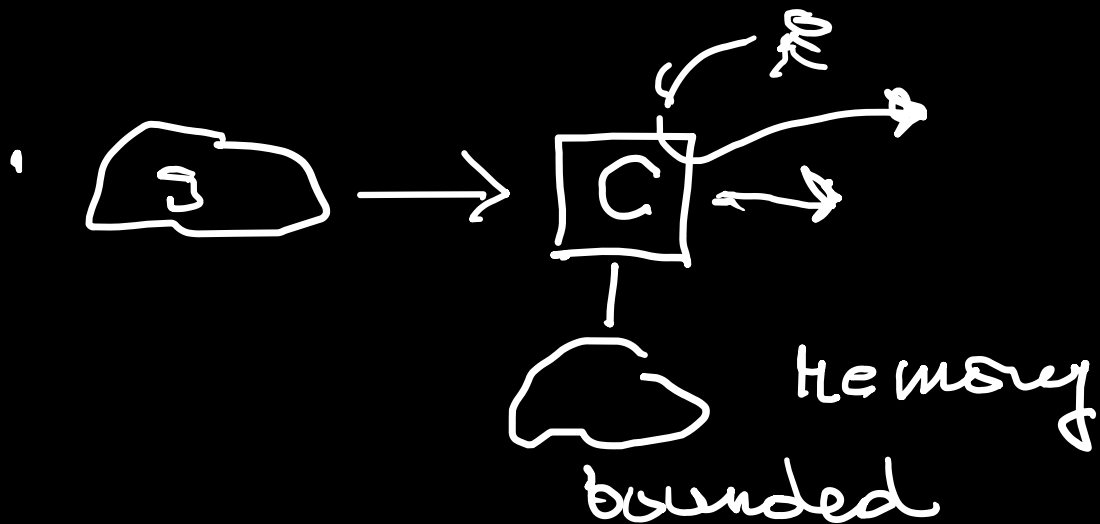
3. wybierz 10 najw.

STREAMING

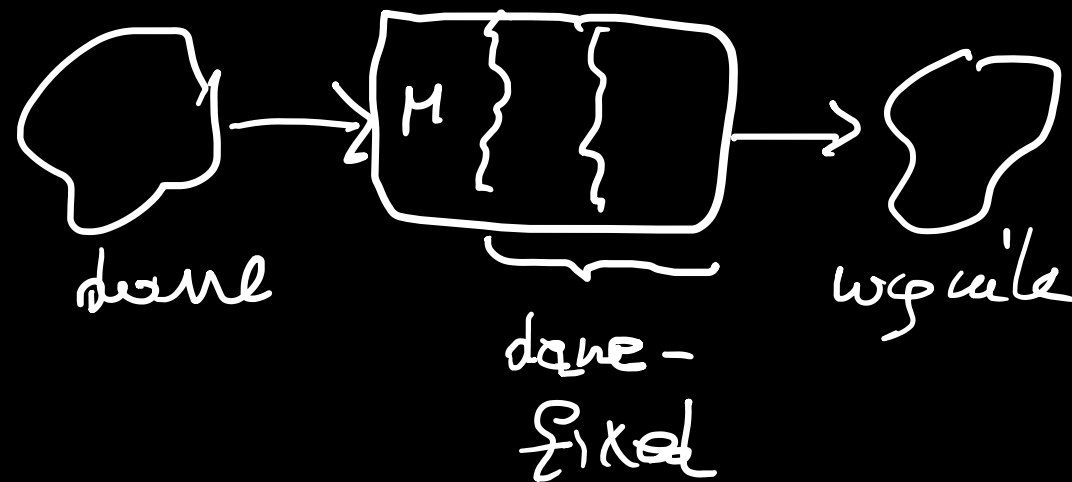
: structured data

- many structured data

$x_1(x_2) \dots x_n$
↑
optional elem



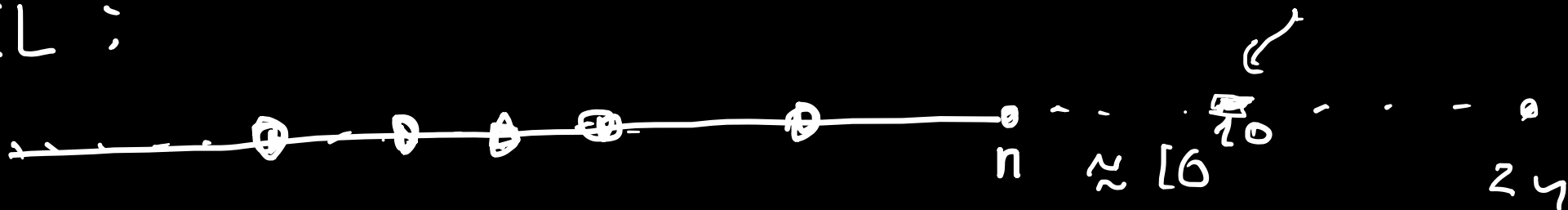
M-R



• šrednia

$$\begin{cases} \text{onInit } (L) \{ L = 0; S = \emptyset \} \\ \text{onGet } (x) \{ L++; S = S + x; \} \\ \text{or Calc } (x) \{ \text{return } (S/L); \} \end{cases}$$

↓ CEL:



↑ ↑ ↑ ↑ ↑

uniform sample

