Kwantyfikatory : II

- ustalone Ω $(\neq \emptyset)$
- funkcje zdaniowe $\varphi, \psi : \Omega \longrightarrow \{0, 1\}$

$A, B \subseteq \Omega$

$A \cap B = \Omega$

$\Updownarrow$

$A = \Omega \wedge B = \Omega$

- $(\forall x)\varphi(x) \equiv (\{\omega \in \Omega : \varphi(\omega)\} = \Omega)$
- $(\exists x)\varphi(x) \equiv (\{\omega \in \Omega : \varphi(\omega)\} \neq \emptyset)$

F1 $(\forall x)(\varphi(x) \wedge \psi(x)) \equiv (\{\omega \in \Omega : \varphi(\omega) \wedge \psi(\omega)\} = \Omega)$

$\equiv (\{\omega \in \Omega : \varphi(\omega)\} \cap \{x \in \Omega : \psi(x)\} = \Omega)$

$\equiv (\{\omega \in \Omega : \varphi(\omega)\} = \Omega) \wedge (\{\omega \in \Omega : \psi(\omega)\} = \Omega) \equiv$

$\equiv (\forall x)\varphi(x) \wedge (\forall x)\psi(x)$

$$\boxed{(\forall x)(\varphi(x) \wedge \psi(x)) \equiv (\forall x)\varphi(x) \wedge (\forall x)\psi(x)}$$

Uwaga: $\Omega = \{\omega_1, \dots, \omega_n\}$

$$(\forall x)\varphi(x) \equiv \varphi(\omega_1) \wedge \dots \wedge \varphi(\omega_n) \qquad (\forall x)\varphi(x) = \bigwedge_{X} \varphi(x)$$

$$\underline{(\forall x)(\varphi(x) \wedge \psi(x))} \equiv (\varphi(\omega_1) \wedge \psi(\omega_1)) \wedge (\varphi(\omega_2) \wedge \psi(\omega_2)) \wedge \dots \wedge (\varphi(\omega_n) \wedge \psi(\omega_n))$$

$$\equiv (\varphi(\omega_1) \wedge \dots \wedge \varphi(\omega_n)) \wedge (\psi(\omega_1) \wedge \dots \wedge \psi(\omega_n))$$

$$\equiv \underline{(\forall x)\varphi(x) \wedge (\forall x)\psi(x)}$$

P

$x/y \cdot z = ?$

$(x/y)\cdot z$

$x/(y\cdot z)$

F2: $((\forall x)\varphi(x) \vee (\forall x)\psi(x)) \longrightarrow (\forall x)(\varphi(x) \vee \psi(x))$

D-d. Możemy zał. że $(\forall x)\varphi(x)$, czyli $\{\omega \in \Omega : \varphi(\omega)\} = \Omega$. Zatem

(!) $\Omega \supseteq \{\omega \in \Omega : \varphi(\omega) \vee \psi(\omega)\} = \underbrace{\{\omega \in \Omega : \varphi(\omega)\}}_{\Omega} \cup \{\omega \in \Omega : \psi(\omega)\} \supseteq \Omega$

$\subseteq \; = \; \supseteq$

$\square$

P. $\Omega = \mathbb{N}$ $\qquad \varphi(x) = \text{"}2 \mid x\text{"}$

$\qquad\qquad\qquad \psi(x) = \text{"}\neg\, 2 \mid x\text{"}$

1. $(\forall x)(\varphi(x) \vee \psi(x)) \equiv T$

2. $(\forall x)\varphi(x) \vee (\forall x)\psi(x) \equiv \bot \vee \bot \equiv \bot$

$$\boxed{\big((\forall x)\varphi(x) \vee (\forall x)\psi(x)\big) \rightarrow (\forall x)(\varphi(x) \vee \psi(x))}.$$

F

$$\left.\boxed{\begin{aligned} \neg(\forall x)\varphi(x) &\equiv (\exists x)(\neg\varphi(x)) \\ \neg(\exists x)\varphi(x) &\equiv (\forall x)(\neg\varphi(x)) \end{aligned}}\ \right\}$$

правило де Моргана за кванторите.

D-d. $\neg (\forall x)\varphi(x) \equiv (\neg(\{\omega\in\Omega : \varphi(\omega)\} = \Omega))$

Ω

$\{\omega\in\Omega : \varphi(\omega)\}$

(1)

$$\equiv \{\omega\in\Omega : \varphi(\omega)\}^c \neq \emptyset$$
$$\equiv \{\omega\in\Omega : \neg\varphi(\omega)\} \neq \emptyset$$
$$\equiv (\exists x)(\neg\varphi(x)) \quad \blacksquare$$

$\models (\alpha\leftrightarrow\beta)\leftrightarrow((\neg\alpha)\leftrightarrow(\neg\beta))$

$\boxed{A\cup B \neq \emptyset \equiv A\neq\emptyset \vee B\neq\emptyset}$

(2) zadanie.

F

$$\boxed{(\exists x)(\varphi(x)\vee\psi(x)) \equiv (\exists x)\varphi(x) \vee (\exists x)\psi(x)}$$

D-d

$$\neg(\exists x)(\varphi(x)\vee\psi(x)) \equiv (\forall x)(\neg(\varphi(x)\vee\psi(x))) \equiv (\forall x)(\neg\varphi(x)\wedge\neg\psi(x))$$
$$\equiv (\forall x)(\neg\varphi(x))\wedge(\forall x)(\neg\psi(x)) \equiv \neg(\exists x)\varphi(x)\wedge\neg(\exists x)\psi(x) \equiv \neg((\exists x)\varphi(x)\vee(\exists x)\psi(x)) \quad \blacksquare$$

(F) $(\exists x)(\varphi(x) \wedge \psi(x)) \longrightarrow ((\exists x)\varphi(x) \wedge (\exists x)\psi(x))$

D-d. $\{\omega \in \Omega : \varphi(\omega) \wedge \psi(\omega)\} \neq \emptyset$

$$\text{IS}$$

$\{\omega \in \Omega : \varphi(\omega)\} \cap \{\omega \in \Omega : \psi(\omega)\} \neq \emptyset$

$A \cap B \neq \emptyset \longrightarrow A \neq \emptyset \wedge B \neq \emptyset$

$$\Downarrow$$

$(\exists x)\varphi(x) \wedge (\exists x)\psi(x)$

(P) $\Omega = \mathbb{N}$; $\varphi(x) = $ "$2|x$" ; $\psi(x) = $ "$\neg 2|x$",

$L \equiv (\exists x)(2|x) \wedge (\exists x)(\neg 2|x) \equiv T \wedge T \equiv T$

$P \equiv (\exists x)(2|x \wedge \neg 2|x) \equiv \perp$

CZYLI: implikacja w drugą stronę nie jest prawdziwa !!!

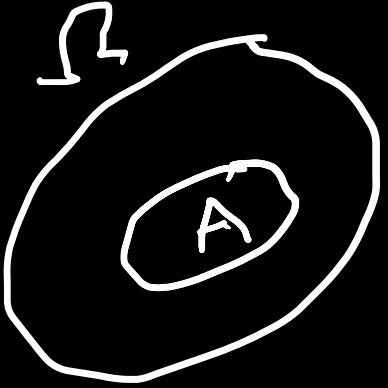
(F) $((\forall x)\,\varphi(x))\wedge\alpha \equiv (\forall x)(\varphi(x)\wedge\alpha)$

nie zależy od x

$((\forall x)\,\varphi(x))\vee\alpha \equiv (\forall x)(\varphi(x)\vee\alpha)$

to samo dla $\exists$

KWANT. OGRANICZONE



Def. $(\forall x\in A)\,\varphi(x) \equiv (\forall x)(x\in A\rightarrow\varphi(x))$

$(\exists x\in A)\,\varphi(x) \equiv (\exists x)(x\in A\wedge\varphi(x))$

(P) $(\forall\varepsilon>0)\ldots\ldots$

$\equiv$

$(\forall\varepsilon\in(0,\infty))\ldots\ldots$

Prawa de Morgana dla kw. ogr.

$$\begin{cases} \neg(\exists x \in A)\,\varphi(x) \equiv (\forall x \in A)(\neg\varphi(x)) \\ \neg(\forall x \in A)\,\varphi(x) \equiv (\exists x \in A)(\neg\varphi(x)) \end{cases}$$

D-d: (2)

$$\neg(\forall x \in A)\varphi(x) \equiv \neg\big((\forall x)(x \in A \to \varphi(x))\big) \equiv$$

$$\equiv (\exists x)(\neg(x \in A \to \varphi(x)))$$

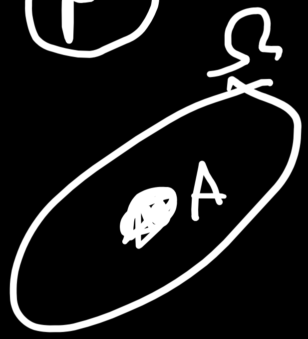
$$\equiv (\exists x)(x \in A \wedge \neg\varphi(x)) \equiv$$

$$\equiv (\exists x \in A)(\neg\varphi(x)) \ \blacksquare$$

$$\begin{cases} \neg(p \to q) \equiv \\ \equiv \neg(\neg p \vee q) \equiv \\ \equiv \neg\neg p \wedge \neg q \equiv \\ \equiv p \wedge \neg q \end{cases}$$

(1) ← zadanie

P



Co się dzieje jeśli $A = \emptyset$?

$$(\forall x \in \emptyset)\,\varphi(x) \equiv (\forall x)(\overbrace{x \in \emptyset}^{\emptyset} \to \varphi(x))$$

$$\equiv T$$

$A =$ "zbiór jednorożców"

$\varphi(x) =$ "x jest pingwinem"

$$(\forall x \in A)(\varphi(x)) \equiv T$$

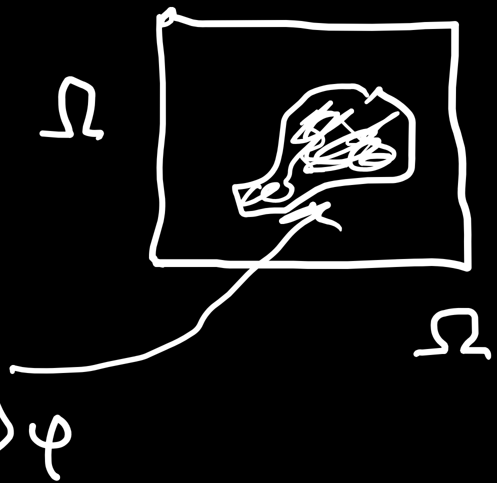
czyli każdy jednorożec jest pingwinem.

DWIE ZMIENNE.

$$\varphi, \psi : \Omega \times \Omega \longrightarrow \{0, 1\}$$

$$D_\varphi = \{(x,y) \in \Omega \times \Omega : \varphi(x,y)\}$$

↑ diagram φ



$$(Q_1 x)(Q_2 y)\, \varphi(x,y) \quad , \quad Q_1, Q_2 \in \forall, \exists$$

$$\circledast \quad (\forall x)\overbrace{(\forall y)\varphi(x,y)}^{\tilde{\varphi}(x)} \overset{\text{def}}{\equiv} (\forall x)\tilde{\varphi}(x),$$

$$\tilde{\varphi}(x) = \text{"}(\forall y)\,\varphi(x,y)\text{"}$$

$$1 \equiv (\forall x)\tilde{\varphi}(x) \equiv \{x \in \Omega : \tilde{\varphi}(x)\} = \Omega$$

$$(1 = \tilde{\varphi}(x)) \equiv \{y \in \Omega : \varphi(x,y)\} = \Omega$$

$$(\forall x)(\forall y)\varphi(x,y)$$

$$\equiv$$

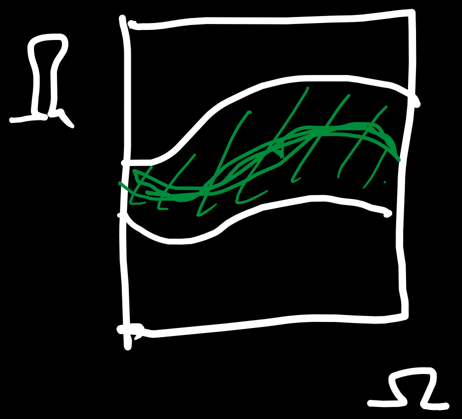
$$\{(x,y) \in \Omega^2 : \varphi(x,y)\} = \Omega^3$$

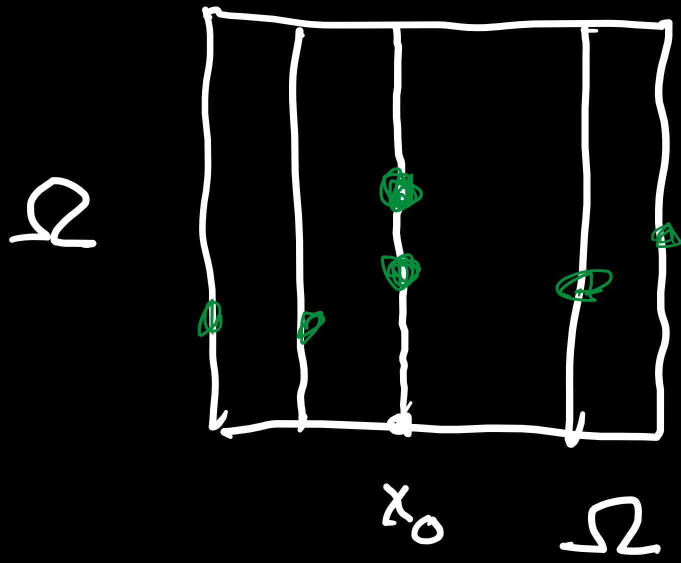
$$\equiv$$

$$D_\varphi = \Omega^2.$$

- $(\forall x)\underbrace{(\exists y)\varphi(x,y)}_{\varphi^*(x)} \equiv (\forall x)\varphi^*(x)$

$$\varphi^*(x) = (\exists y)\varphi(x,y)$$

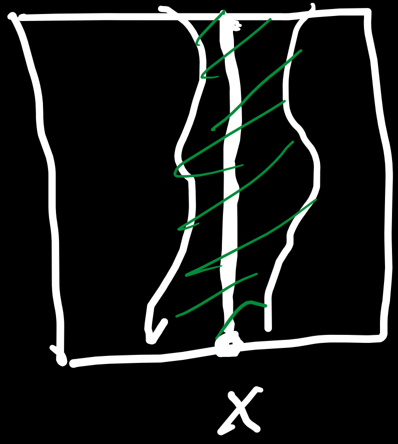




- $(\exists x)(\forall y)\varphi(x,y) \equiv$

$$\tilde{\varphi}(x) = (\forall y)\varphi(x,y)$$

$$\equiv (\exists x)\tilde{\varphi}(x)$$



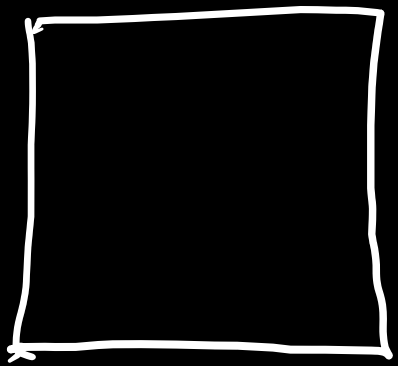
• $(\exists x)(\exists y)\varphi(x,y)$

$\equiv$

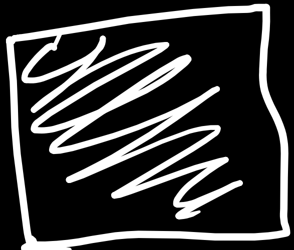
$\{(x,y)\in\Omega^2 : \varphi(x,y)\} \neq \emptyset$

$\equiv$

$D_\varphi \neq \emptyset$

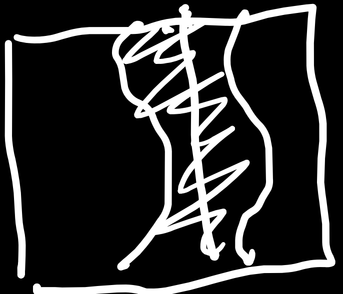


$D_\varphi = \Omega^2$



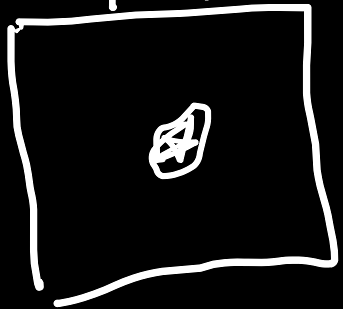
$(\forall x)(\forall y)\varphi(x,y)$

$\forall x \exists y\, \varphi(x,y)$



$\exists x\, \forall y\, \varphi(x,y)$

$D_\varphi \neq \emptyset$



$(\exists x)(\exists y)\varphi(x,y)$

$\boxed{\Omega \neq \emptyset}$

$(A\times B)\times C = \{((a,b),c) : a\in A \wedge b\in B \wedge c\in C\}$

Po wykładzie

$(a,b,c)_1 = ((a,b),c)$

$= \{(a,b,c) : a\in A \wedge b\in B \wedge c\in C\}$

$A\times(B\times C) = \{(a,(b,c)) : \text{——} \wedge \text{——}\}$

$(a,b,c)_2 = (a,(b,c))$

Teorie kategori

$\varphi : (A\times B)\times C \xrightarrow[\text{na}]{1-1} A\times(B\times C)$

$\varphi(\underbrace{((a,b),c)}) = \underbrace{(a,(b,c))}$

$\varphi((a,b,c)_1) = \varphi((a,b,c)_2)$

$$p \oplus (q \oplus r) \equiv p \oplus \underbrace{((q \wedge \neg r) \vee (\neg q \wedge r))}_{\alpha} \equiv (p \wedge \neg \alpha) \vee (\neg p \wedge \alpha)$$

$$\equiv (p \wedge ((\neg q \vee r) \wedge (q \vee \neg r))) \vee (\neg p \wedge (\ldots)) \equiv$$

Tabelka 0 – 1

p	q	r	$q \oplus r$	$(p \oplus q)$	L	P
1	1	1				
1	1	0				
1	0	1				
1	0	0				
0	.	.				
0		.				
0		.				
0		.				

$\alpha \equiv \bot$

$$\frac{p, \neg p \vee \beta}{\beta} \quad p \to \beta$$

$$\frac{p, p \to \beta}{\beta} \quad MP$$

{1 mówi że 2 mamy

$$\frac{p \vee \alpha, \neg p \vee \beta}{\alpha \vee \beta}$$

rezolucja