- $f: X \to Y$; $x \sim_f y \overset{\text{def}}{\equiv} f(x) = f(y)$

  $\sim_f$ – rel. równ. na X

- $\approx$ rel. równ. na X ; $\varphi(x) = [x]_{\approx}$ $(x \in X)$; czyli

  $\varphi = \{(x, [x]_{\approx}) : x \in X\}$

  $\varphi : X \to X/\approx$

  wtedy: $x \sim_\varphi y \equiv \varphi(x) = \varphi(y) \equiv [x]_{\approx} = [y]_{\approx}$

  $\equiv x \approx y$

  czyli : $\boxed{\sim_\varphi = \approx}$

- $X/\approx$ – rozbicie zbioru X

  $\bigcup(X/\approx) = X$

- $\mathcal{P}$ jest rozbiciem (partycją) zbioru X
    - $\bigcup \mathcal{P} = X$
    - $A \in \mathcal{P} \to A \neq \emptyset$
    - $A, B \in \mathcal{P} \wedge A \cap B \neq \emptyset \longrightarrow A = B$

      (równ.: $A \neq B \longrightarrow A \cap B = \emptyset$)

na X definiujemy

$$x \simeq y \equiv (\exists A \in \mathcal{P})(\{x, y\} \subseteq A)$$

- $x \simeq x \equiv (\exists A \in \mathcal{P})(\{x, x\} \subseteq A) \equiv$
  $\equiv (\exists A \in \mathcal{P})(x \in A) \equiv x \in \bigcup \mathcal{P}$.
- $x \simeq y \equiv (\exists A \in \mathcal{P})(\{x, y\} \subseteq A) \equiv$
  $\equiv (\exists A \in \mathcal{P})(\{y, x\} \subseteq A) \equiv y \simeq x$

Zał. że $x \approx y$ i $y \approx z$.

mamy $A \in \mathcal{P}$ t. że $\{x, y\} \subseteq A$.

mamy $B \in \mathcal{P}$ t. że $\{y, z\} \subseteq B$.

wtedy $y \in A$ i $y \in B$, więc $A \cap B \neq \emptyset$

więc $A = B$. (bo $\mathcal{P}$ jest rozbiciem)

wtedy $\{x, y\} \subseteq A$, $\{y, z\} \subseteq A$

wtedy $\{x, z\} \subseteq A$ więc $x \approx z$

przechodniość.

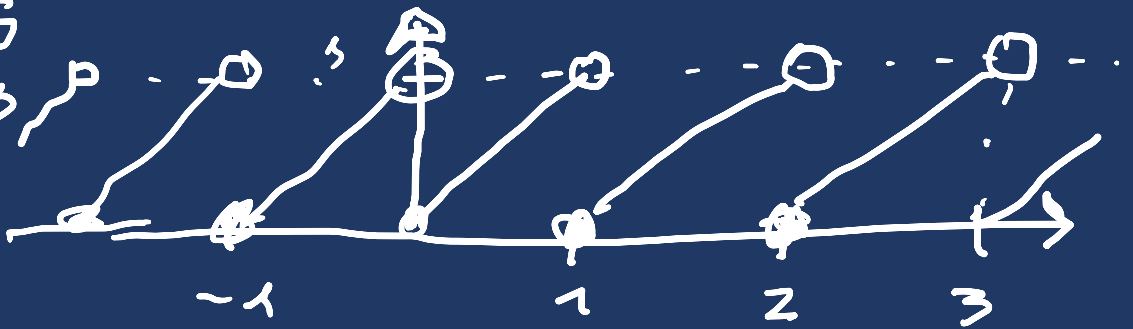
$\mathcal{P}$ - rozb $\Rightarrow \left(x \approx_{\mathcal{P}} y \equiv (\exists A \in \mathcal{P})(\{x, y\} \subseteq A)\right) \leftarrow$ rel. równoważności

Z FAKT: $X / \approx_{\mathcal{P}} = \mathcal{P}$.

P

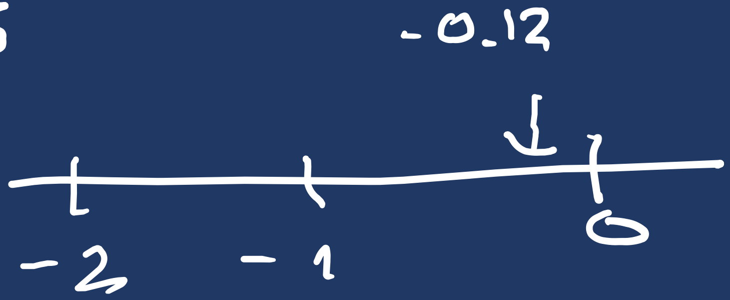
$$fr : \mathbb{R} \to \mathbb{R} : x \to x - \lfloor x \rfloor$$

$$\begin{cases} fr(1.25) = 0.25 \\ fr(0.13) = 0.13 \\ fr(3) = 0 \end{cases}$$



$$\lfloor x \rfloor = \max\{k \in \mathbb{Z} : k \leq x\}$$

$$\lfloor -0.12 \rfloor = -1$$



$$fr(-0.12) = -0{,}12 + 1$$

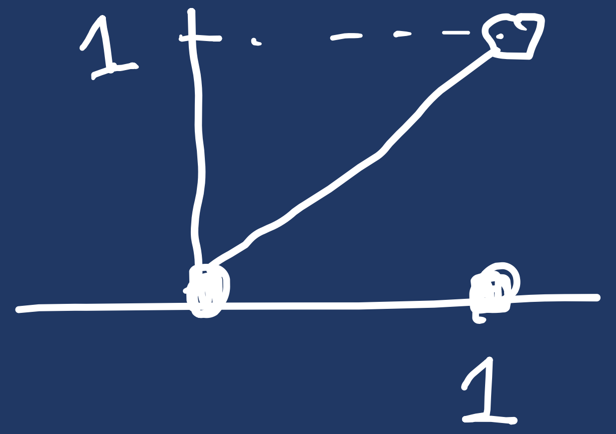
$$= 0.88.$$

$$\lfloor x \rfloor = -\lceil -x \rceil$$

$$\lceil x \rceil = \min\{k \in \mathbb{Z} : x \leq k\}$$

$X = [0,1]$

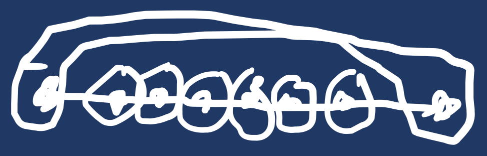
$\varphi = \mathrm{fr}\restriction_{[0,1]}$



$x,y \in [0,1]$ $\quad x \sim_\varphi y \equiv \mathrm{fr}(x) = \mathrm{fr}(y)$

- $0 \sim_\varphi 1$
- $0 < x < 1$ $\quad 0 \sim_\varphi x \leftarrow$ nie

$0=\mathrm{fr}(0)$; $\mathrm{fr}(x) = x \neq 0$



- $0 < x,y < 1$!
- $[0] = \{0,1\}$
- $0 < x < 1$; $[x] = \{x\}$

$x \sim_\varphi y \equiv x = y$;

$\{0,1\}$

$[0,1]/\sim_\varphi$

(P)

$X = [0,1]^2$

$(x,y) \sim (x',y') \equiv (\mathrm{fr}(x) = \mathrm{fr}(x') \wedge y = y')$

$\varphi : [0,1]^2 \to \mathbb{R}^2 \; ; \; \varphi((x,y)) = (\mathrm{fr}(x), y)$

$\sim \; = \; \sim_\varphi .$

$$\varphi((x,y)) = (\mathrm{fr}(x), y)$$

$/\sim_\varphi \;\cong$

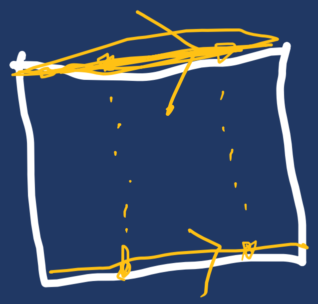
sklejamy

walec

(P) $\psi : [0,1]^2 \to \mathbb{R} : \psi((x,y)) = (\mathrm{fr}(x), \mathrm{fr}(y))$

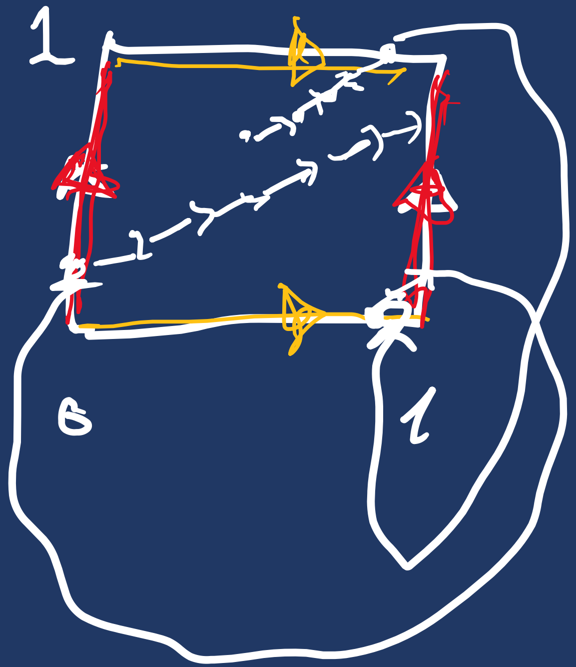
$(x,y) \sim_\psi (x',y')$

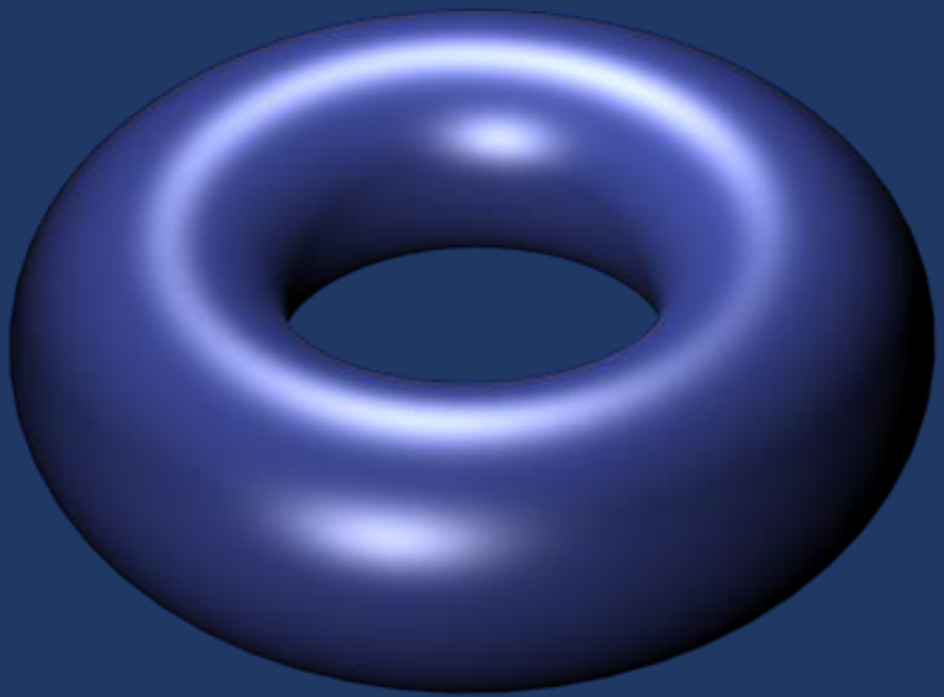
$(x,0) \sim_\psi (x,1)$



pierwsze zlepienie

drugie zlepienie

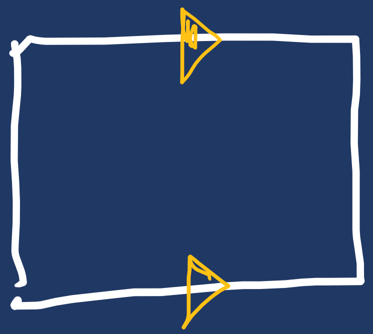


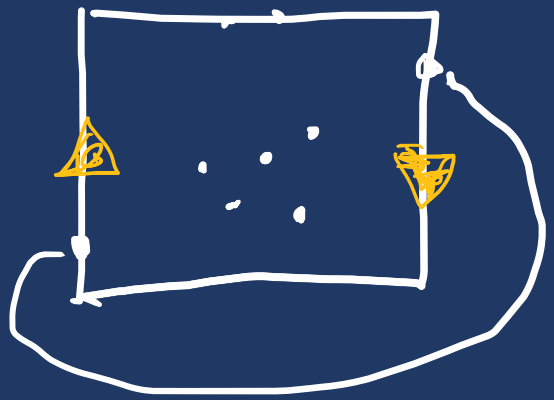


TORUS

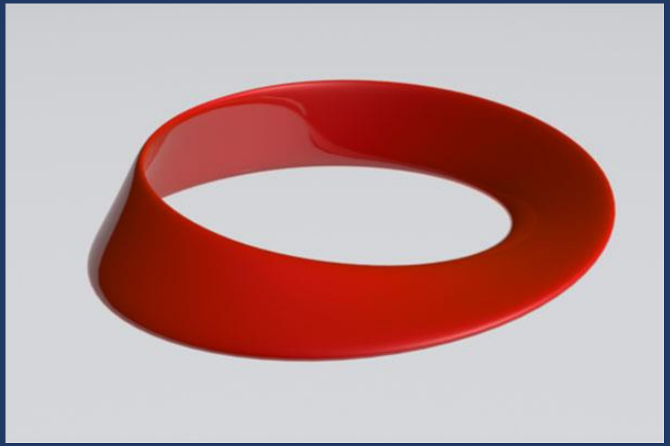
świat torusa

P





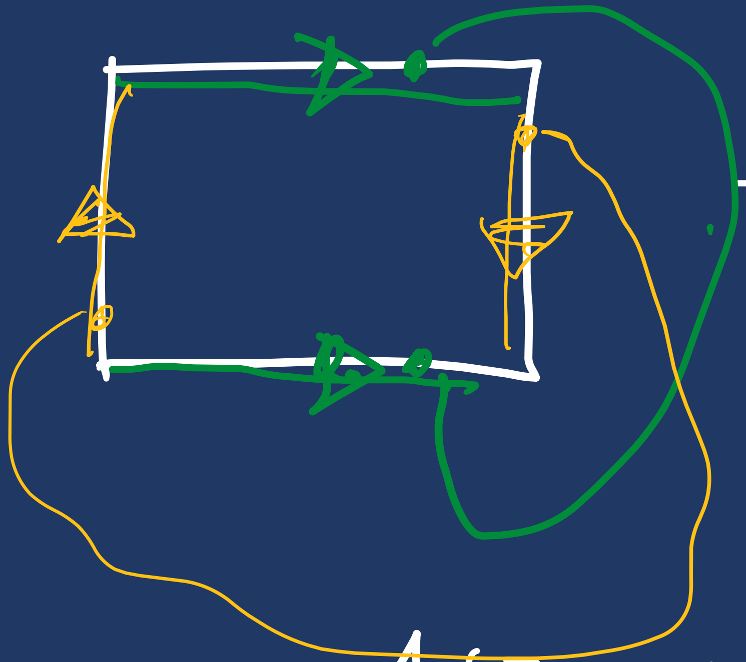
$$\mathcal{P} = \{\{(0,y),(1,1-y)\} : y \in [0,1]\}$$
$$\cup \{\{(x,y),(x,y)\} : x,y \in [0,1]\}$$



Wstęga Möbiusa

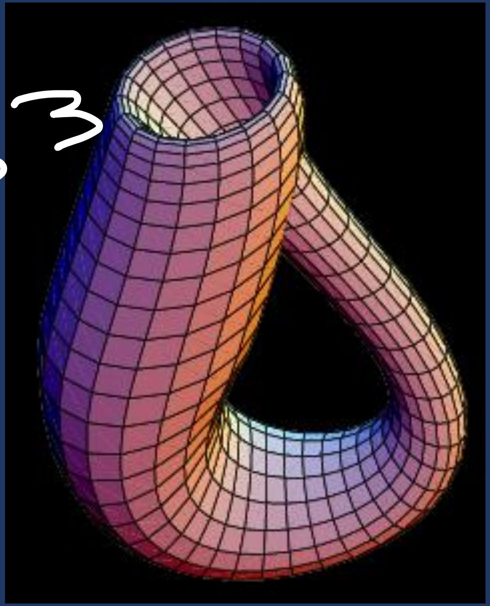
powierzchnia jednostronna

P



Butelka Kleina

tego się da zrobić w $\mathbb{R}^3$



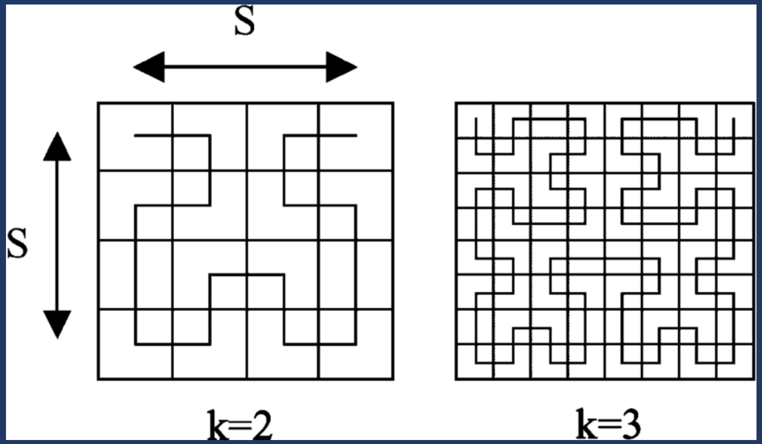
ALE można w $\mathbb{R}^4$

homeomorfizm

krzywe Peano

$f: [0,1] \xrightarrow[\text{na}]{\text{ciąg}} [0,1]^2$

$$f_0 : [0,1] \to [0,1]^2$$



krzywe Hilberta

f_1

$$f^*(t) = \lim_n f_n(t)$$

Krzywe Peano

$$f : [0,1] \xrightarrow{\text{na}} [0,1]^2$$

ANALIZA II

ALGEBRAICZNE KONSTRUKCJE

$\mathcal{G} = (G, +)$ - grupa

$H \subseteq G$: jest podgrupą

'jeśli' 1) $H \neq \emptyset$

2) $x, y \in H \to x + y \in H$

3) $x \in H \to -x \in H$

$(2) \wedge (3) \equiv (x, y \in H \to x - y \in H)$

$\mathcal{G} = (\mathbb{Z}, +)$

$H = \{5k : k \in \mathbb{Z}\}$

- $5k + 5l = 5 \cdot (k + l)$
- $-(5 \cdot k) = 5 \cdot (-k)$

H - podgr. grupy $G = (G, +)$. ← abelowa

$$x + y = y + x$$

Na G określamy

$$x \sim_H y \equiv x - y \in H$$

P $(\mathbb{Z}, +)$

$$H = \{5k : k \in \mathbb{Z}\}$$

$$x \sim y \equiv x - y \in H$$

$$\equiv 5 \mid (x - y)$$

- $x \sim x \equiv x - x \in H \equiv 0 \in H$

  sprawdź to

- $x \sim y \equiv x - y \in H \rightarrow -(x-y) \in H$

  $\rightarrow y - x \in H$

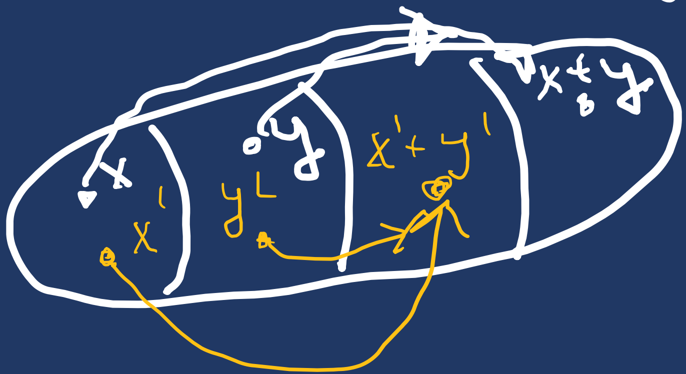
- $\left.\begin{matrix} x \sim y \\ y \sim z \end{matrix}\right\} \equiv \left.\begin{matrix} x - y \in H \\ y - z \in H \end{matrix}\right\} \Rightarrow (x-y) + (y-z) \in H$

  $\Rightarrow x - z \in H \rightarrow x \sim z$

$\sim_H$ - rel. równ. na G

$$G/\sim_H \;=\; \{[g]_{\sim_H} : g \in G\}$$

$$? \quad [x]_\sim + [y]_\sim = [x+y]_\sim$$



G

Poprawność:

$$\left.\begin{array}{l} x \sim x' \\ y \sim y' \end{array}\right] \to (x+y) \sim (x'+y')$$

$$x - x' = x + (-x')$$

D-d

$$\left\{\begin{array}{l} x \sim x' \\ y \sim y' \end{array}\right\} \equiv \left\{\begin{array}{l} x - x' \in H \\ y - y' \in H \end{array}\right\} \Rightarrow (x-x') + (y-y') \in H$$

$$x + (-x') + (y) + (-y')$$

$$x + y \sim (x'+y')$$

$$\Rightarrow (x+y) + ((-x') + (-y')) \in H \to (x+y) - (x'+y') \in H$$

(P) $(\mathbb{Z}, +)$ $\quad 5\mathbb{Z} = \{5 \cdot k : k \in \mathbb{Z}\}$

$$[3]_\sim + [4]_\sim = [3+4]_\sim = [2]_\sim$$

$7 \sim 2$

← elem. neutralny

$$[3]_\sim + [0]_\sim = [3+0]_\sim = [3]_\sim$$

$$G/H = (G/_{\sim_H}, +_H)$$

$$\mathbb{Z}/5\mathbb{Z} = \mathbb{C}_5$$

+ mod 5