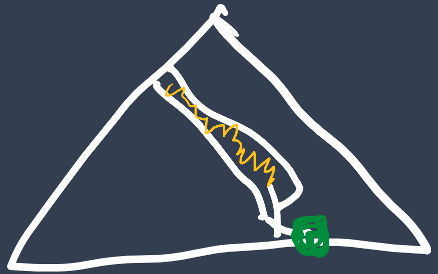
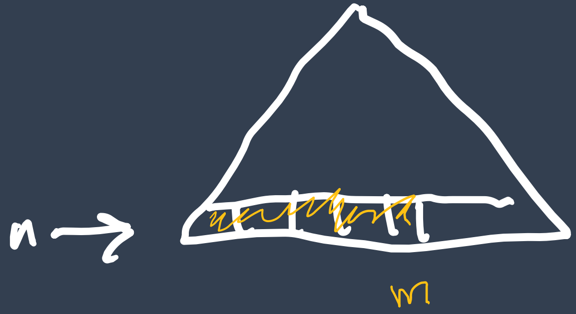
Współcz. dwumianowe – c.d.

1

$$\sum_{k\le n}\binom{k}{a}=\binom{n+1}{a+1}$$



$$\sum_{k\le n}\binom{a+k}{k}=\binom{a+n+1}{n}$$



$$\sum_{k\le m}(-1)^k\binom{n}{k}=(-1)^m\binom{n-1}{m}$$

nie znamy zwartej postaci na

ustalone $n \in \mathbb{N}$: $\sum_{k \le m} \binom{n}{k}$

$\binom{n}{k}$

znamy tylko aproksymacje:

$m \ll n/2$ $\qquad \sum \approx \dots$

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rozkł. prawd.

do tego wrócimy później: CLT

2) Ogólnienie

$$\binom{n}{3} = \frac{n!}{3!\,(n-3)!} = \frac{\cancel{(n-3)!}(n-2)(n-1)\,n}{3!\,\cancel{(n-3)!}}$$

$$= \frac{n(n-1)(n-2)}{3!}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n(n-1)\cdot\ldots\cdot(n-k+1)}{k!}$$

Def. $x \in \mathbb{C}$, $k > 0$, $k \in \mathbb{N}$

dolna silnia

$$x^{\underline{k}} = \prod_{l=0}^{k-1} (x-l) \qquad x^{\underline{0}} = 1$$

np. $x^{\underline{3}} = (x-0)(x-1)(x-2)$

górna silnia

$$x^{\overline{k}} = \prod_{l=0}^{k-1} (x+l) \qquad x^{\overline{0}} = 1$$

$$x^{\overline{3}} = x(x+1)(x+2)$$

DEF. Dla $x \in \mathbb{C}$ i $k \in \mathbb{N}$

$$\binom{x}{k} = \frac{x^{\underline{k}}}{k!}$$

Uwaga:

- $x^{\underline{0}} = x^{\overline{0}} = 1$

P: $$\binom{-1}{k} = \frac{1}{k!} \prod_{l=0}^{k-1} (-1-l) =$$

$$\frac{1}{k!} (-1)^k \prod_{l=0}^{k-1} (1+l) = \frac{1}{k!} (-1)^k \cdot k! = (-1)^k$$

Uwaga:

$$\binom{x}{k} = \frac{x^{\underline{k}}}{k!} = \frac{1}{k!}\underbrace{\prod_{l=0}^{k-1}(x-l)}$$

liczba naturalna

wielomian zmiennej x stopnia k

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wniosek

Arg. wielom.

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$k \geq 1$; $n \geq 1$

TW

$$\boxed{\underbrace{\binom{x}{k}}_{L(x)} = \underbrace{\binom{x-1}{k} + \binom{x-1}{k-1}}_{P(x)}}$$

$(\forall x)$

L, P - wielomiany st. k

$L(k) = P(k)$, $L(k+1) = P(k+1)$

$L(k+m) = P(k+m)$, $m \geq 0$

$(\forall x)\ P(x) = L(x)$

Pochodne

1. $(x^{\alpha})' = \alpha x^{\alpha-1}$

$$(x^{\alpha})'' = \alpha(\alpha-1)x^{\alpha-2}$$

$$(x^{\alpha})^{(k)} = \alpha^{\underline{k}} \cdot x^{\alpha-k}$$

2. $\left((1+x)^{\alpha}\right)^{(k)} = \alpha^{\underline{k}}(1+x)^{\alpha-k}$

Konwergja: $f^{(0)} = f.$

$$f(x) = (1+x)^{\alpha} \qquad x, \alpha \in \mathbb{R}$$

wzór McLaurina (Taylora)

$$f(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{k!} x^k + \underbrace{\frac{f^{(n)}(\theta_x x)}{n!} x^n}_{R_n(x)}$$

$$= \sum_{k=0}^{n-1} \frac{\alpha^{\underline{k}}}{k!} x^k + R_n(x)$$

$$= \sum_{k=0}^{n-1} \binom{\alpha}{k} x^k + R_n(x)$$

FAKT:

$$|x| < 1 \rightarrow \lim_{n} R_n(x) = 0$$

$$\boxed{(1+x)^{\alpha} = \sum_{n\geq 0} \binom{\alpha}{n} x^n, \quad |x|<1}$$

uogólniona postać wzoru dwumianu.

P

$$(1+x)^{-1} = \sum_{k\geq 0} \binom{-1}{k} x^k = \sum_{k\geq 0} (-1)^k x^k$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots, \quad |x|<1$$

$$1+x+\ldots+x^k = \frac{1-x^{k+1}}{1-x}$$