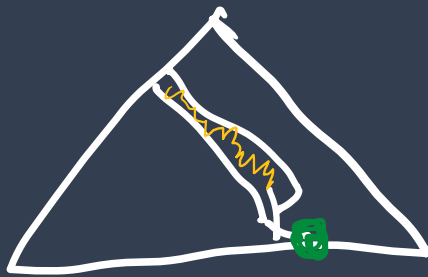


Współcz. dwumianowe $\sim \triangleleft d$,

①



$$\sum_{k \leq n} \binom{k}{a} = \binom{n+1}{a+1}$$



$$\sum_{k \leq n} \binom{a+k}{k} = \binom{a+n+1}{n}$$



$$\sum_{k \leq m} (-1)^k \binom{n}{k} = (-1)^m \binom{n-1}{m}$$

Nie znamy zera tej postaci na

ustalone $n \in \mathbb{N}$; $\sum_{k \leq m} \binom{n}{k} \binom{m}{k}$

The diagram shows a horizontal bar divided into segments. The top part is labeled with 'n' and 'm'. The bottom part is labeled with 'k' and 'm'. An arrow points from the binomial coefficient expression to the diagram.

Znamy tylko odpowiednio:

$$m \ll n/2$$

$$\sum \approx \dots$$

$$m \approx n/2$$

$$\sum \approx \dots$$

→ wada prawa.

do tego wrócimy później:

CLT

2) Ogó dwiukrotnie

$$\binom{n}{3} = \frac{n!}{3!(n-3)!} = \frac{\cancel{(n-3)!} (n-2)(n-1)n}{3! \cancel{(n-3)!}}$$
$$= \frac{n(n-1)(n-2)}{3!}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\dots(n-k+1)}{k!}$$

Def. $x \in \mathbb{C}$, $k > 0$, $k \in \mathbb{N}$

dolna silnia

$$x^{\underline{k}} = \prod_{l=0}^{k-1} (x-l)$$

$$x^{\underline{0}} = 1$$

np. $x^{\underline{3}} = (x-0)(x-1)(x-2)$

gorna silnia

$$x^{\overline{k}} = \prod_{l=0}^{k-1} (x+l)$$

$$x^{\overline{0}} = 1$$

$$x^{\overline{3}} = x(x+1)(x+2)$$

DEF. Dla $x \in \mathbb{C}$ i $k \in \mathbb{N}$

$$\binom{x}{k} = \frac{x^{\underline{k}}}{k!}$$

Uwaga:

$$x^{\underline{0}} = x^{\overline{0}} = 1$$

$$P: \binom{-1}{k} = \frac{1}{k!} \prod_{l=0}^{k-1} (-1-l) =$$

$$\frac{1}{k!} (-1)^k \prod_{l=0}^{k-1} (1+l) = \frac{1}{k!} (-1)^k \cdot k! = (-1)^k$$

Wzrost $\binom{x}{k}$:

$$\binom{x}{k} = \frac{x^{\overbrace{k}}}{k!} = \frac{1}{k!} \prod_{l=0}^{k-1} (x-l)$$

liczby naturalne

wielomian zmiennej x stopnia k

Wzrost $\binom{x}{k}$

Ang. wielom.

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$$k \geq 1; n \geq 1$$

L, P - wielomiany st. k

$$L(k) = P(k), L(k+1) = P(k+1)$$

$$L(k+m) = P(k+m) \quad m \geq 0$$

$$(\forall x) P(x) = L(x)$$

Tw

$$\binom{x}{k} = \binom{x-1}{k} + \binom{x-1}{k-1}$$

$L(x)$

$P(x)$

$(\forall x)$

Решение ① $(x^\alpha)^L = \alpha x^{\alpha-1}$

$$(x^\alpha)^{LL} = \alpha(\alpha-1)x^{\alpha-2}$$

$$(x^\alpha)^{(k)} = \alpha \underline{k} \cdot x^{\alpha-k}$$

$$\textcircled{2} \left((1+x)^\alpha \right)^{(k)} = \alpha \underline{k} (1+x)^{\alpha-k}$$

конверсия: $f^{(0)} = f.$

$$f(x) = (1+x)^\alpha \quad x, \alpha \in \mathbb{R}$$

wzór McLaurina (Taylora)

$$f(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{k!} x^k + \underbrace{\frac{f^{(n)}(\theta_x)}{n!} x^n}_{R_n(x)}$$

$$= \sum_{k=0}^{n-1} \frac{\alpha^{\overline{k}}}{k!} x^k + R_n(x)$$

$$= \sum_{k=0}^{n-1} \binom{\alpha}{k} x^k + R_n(x)$$

FAKT:

$$|x| < 1 \rightarrow \lim_n R_n(x) = 0$$

$$(1+x)^{\alpha} = \sum_{n \geq 0} \binom{\alpha}{n} x^n, \quad |x| < 1$$



(P)

ogólniona postać wzoru dwuczłowego.

$$(1+x)^{-1} = \sum_{k \geq 0} \binom{-1}{k} x^k = \sum_{k \geq 0} (-1)^k x^k$$

$$\frac{1}{1+x}$$

$$= 1 - x + x^2 - x^3 + x^4 - \dots$$

$$|x| < 1$$

$$1 + x + \dots + x^k = \frac{1 - x^{k+1}}{1 - x}$$