Wzór Eulera dla "psl"

$$\varphi(n) = n \prod_{\substack{p \mid n \\ p \in PRIME}} \left(1 - \frac{1}{p}\right)$$

konwencja z teorii liczb

"$p \mid n$" $\equiv$ $p \mid n \wedge p \in PRIME$

Współczynniki Newtona (dwumianowe)

$$\binom{n}{k} = \left| \{ T \subseteq [n] : |T| = k \} \right|.$$

gdzie $[n] = \{1, 2, \ldots, n\}$

uwagi: 1) $[0] = \emptyset$

$$\binom{0}{k} = \begin{cases} 1 : k = 0 \\ 0 : k > 0 \end{cases}$$

2) $k \geq 0$ ← zawsze

3) $\left.\begin{matrix} n \in \mathbb{N} \\ k > n \end{matrix}\right] \rightarrow \binom{n}{k} = 0.$

CO WIEMY:

- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ $\quad 0 \leq k \leq n$

$= \frac{n^{\underline{k}}}{k!}$.

$x^{\underline{k}} = \prod_{i=0}^{k-1} (x-i)$

$= x(x-1)\cdot\ldots\cdot(x-k+1)$

- $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

$\underbrace{P([n])}_{2^n} = \bigcup_{k=0}^{n} \underbrace{\{T \subseteq [n] : |T| = k\}}_{\binom{n}{k}}$

- $$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{n-k} = \frac{n!}{(n-k)!\,(n-(n-k))!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$$

WN. $0 \leq k \leq n \longrightarrow \binom{n}{k} = \binom{n}{n-k}$.

- Tożsamość Pascala

$\binom{n+1}{k+1}$

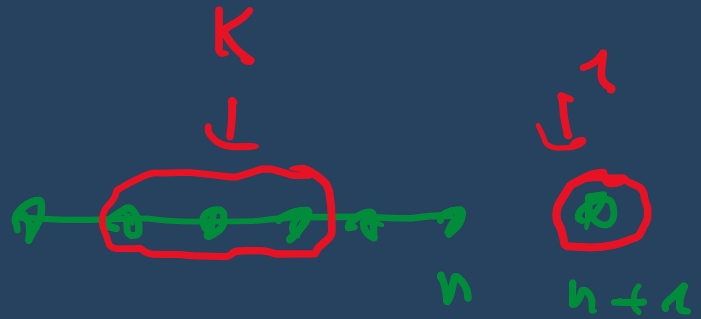
$\binom{n}{k+1}$

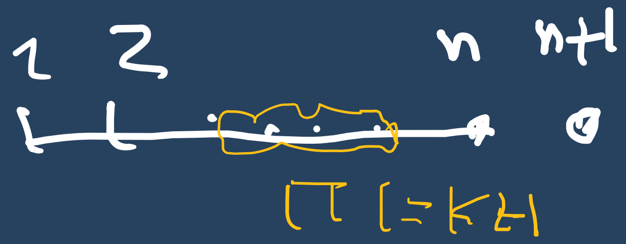
$$\{T \subseteq [n+1] : |T| = k+1\} =$$

$$= \{T \subseteq [n+1] : |T| = k+1 \wedge n+1 \notin T\} \,\dot{\cup}$$

$$\{T \subseteq [n+1] : |T| = k+1 \wedge n+1 \in T\}$$

$$\{T \cup \{n+1\} : T \subseteq [n] \wedge |T| = k\} \leftarrow \binom{n}{k}$$





$$\boxed{\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}}$$

- $\binom{n}{0} = 1$, $\binom{n}{n} = 1$; $n \in \mathbb{N}$

$$\binom{2}{1} = \binom{1+1}{0+1} = \binom{1}{1} + \binom{1}{0}$$

Trójkąt Pascala

$$\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3}
\end{array}$$

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1
\end{array}$$

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

. . . .

$$\binom{n}{1} = \frac{n!}{1!\,(n-1)!} = n$$

								Σ
0:	1	0	0	0	0	0		$\to$ 1
1:	1	1	0	0	0	0		$\to$ 2
2:	1	2	1	0	0	0		$\to$ 4
3:	1	3	3	1	0	0		$\to$ 8
4:	1	4	6	4	1	0		$\to$ 16
5:	1	5	10	10	5	1		$\to$ 32
	1	6	15	20	15	6	1	$\to$ 64

$$\sum_{k \le n} \binom{k}{1} = \binom{n+1}{2}$$

$$\boxed{\sum_{k\le n}\binom{k}{a}=\binom{n+1}{a+1}}$$

TW.

D-d ① algebr. (F(X a)

$n=a$: $\binom{a}{a}=1=\binom{a+1}{a+1}$

zał. ind.

$$\sum_{k\le n+1}\binom{k}{a}=\sum_{k\le n}\binom{k}{a}+\binom{n+1}{a}=$$

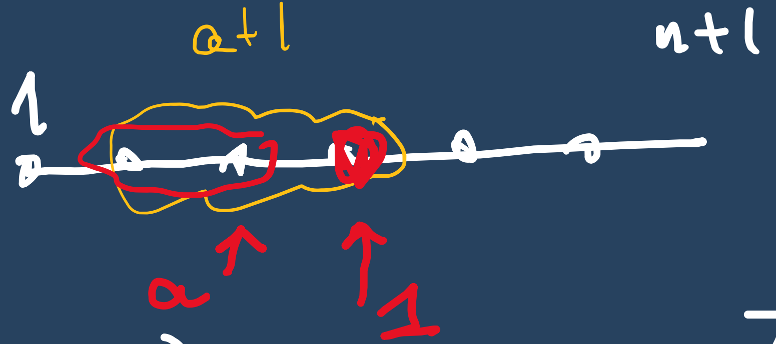
$$=\binom{n+1}{a+1}+\binom{n+1}{a}=\binom{n+2}{a+1}=\binom{(n+1)+1}{a+1}\quad\boxtimes$$

TOŻ. PASC

(2) kombinatorycznie

$$\binom{n+1}{a+1} = \left|\left\{ T \subseteq [n+1] : |T| = a+1 \right\}\right|$$

$$= \left| \bigcup_{k=0}^{n+1} \left\{ T \subseteq [n+1] : |T| = a+1 \wedge \max(T) = k \right\} \right|$$



$$= \left| \dot{\bigcup_{k=\cancel{0}1}^{n+1}} \left\{ T \cup \{k\} : T \subseteq \{1, \dots, k-1\} \wedge |T| = a \right\} \right|$$

$$= \sum_{k=1}^{n+1} \binom{k-1}{a} \underset{l=k-1}{=} \sum_{l=0}^{n} \binom{l}{a}.$$

$$\left(= \sum_{l=a}^{n} \binom{l}{a} \right).$$

$$\left[\text{BO:} \; l < a : \binom{l}{a} = 0 \right]$$

- Jak obliczać $\binom{n}{k}$?
  - ? Tożs. Pascala : zadanie

LIPA

▷ $0 < k \leq n$:

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n}{k} \cdot \frac{(n-1)!}{(k-1)!\,(n-k)!}$$

$$= \frac{n}{k} \, \frac{(n-1)!}{(k-1)!\,\big((n-1)-(k-1)\big)!}$$

$$= \frac{n}{k} \binom{n-1}{k-1}$$

$$\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}.$$

Zadanie:
Intr. w △ Pascala

W ALGOR: skorzystaj z symetrii $\binom{n}{k} = \binom{n}{n-k}$

WZÓR DWUMIANOWY NEWTONA

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}. \qquad n \in \mathbb{N} \quad x, y \in \mathbb{C}$$

D-d: (klasyczny)

- $n=1$: ok
- ind. na n:

$$(x+y)^{n+1} = (x+y)^n (x+y) = \left(\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}\right)(x+y)$$

$$= \underset{\text{toż. Pascala}}{\ldots\ldots} = \ldots\ldots \quad \blacksquare$$

ZADANIE:
zakończcie to

Bew (alg-komb).

$$\prod_{i=1}^{n}(x_i+y_i) = (x_1+y_1)(x_2+y_2)\cdot\ldots\cdot(x_{n-1}+y_{n-1})\cdot(x_n+y_n)$$

$$= \sum_{T\subseteq[n]} \prod_{i\in T} x_i \cdot \prod_{i\in[n]\setminus T} y_i \, .$$

$$(x+y)^n = \sum_{T\subseteq[n]} x^{|T|}\cdot y^{n-|T|} = \sum_{k=0}^{n}\sum_{T\in[n]^k} x^k y^{n-k}$$

$$= \sum_{k=0}^{n}\binom{n}{k} x^k y^{n-k} \, .$$

~~Uwaga!~~ $\cancel{(x+y)^0 = 1}$ bez sensu

OZNACZENIE:

φ – zdanie

$$[\![\varphi]\!] = \begin{cases} 1 & : \varphi \\ 0 & : \neg\varphi \end{cases}$$

nawias logiczny

konwersji BOOL na INT.

wn. z toż. dwum.

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k.$$

- $(1+1)^n = \sum_{k=0}^{n} \binom{n}{k}$

$\quad\; \shortparallel$

$\quad 2^n$

- $(1-1)^n = 0^n = [\![n=0]\!]$

$$[\![n=0]\!] = \sum_{k=0}^{n} \binom{n}{k} (-1)^k$$

!!!

$$1 \quad \longrightarrow \quad 1$$

$$1 \quad 1 \quad \longrightarrow \quad 0$$

$$1 \quad 2 \quad 1 \quad \longrightarrow \quad 0$$

чер. знаки

$$1 \quad 3 \quad 3 \quad 1 \quad \longrightarrow \quad 0$$

$$1 \quad 4 \quad 6 \quad 4 \quad 1 \quad \longrightarrow \quad 0$$

1

1 1

1 2 1

1 3 3 1 0 0

1 4 6 4 1 0

5: 1 5 10 10 5 1

TW

zadanie: zrób to ind. po "a" wzór Pascala

$$\sum_{k \le a} \binom{n}{k} (-1)^k \quad : \quad n=5: \; 1, -4, 6, -4, 1, 0$$

HIPOTEZA: $$\sum_{k \le a} \binom{n}{k} (-1)^k = \binom{n-1}{a} (-1)^a$$

$$\sum_{k \leq a} \binom{n}{k} = ?$$

NIE MA ZNANEJ ROZSĄDNEJ FORMUŁY !!!

$$\sum_{k=0}^{n} \binom{n}{k} \cdot k = \sum_{k=1}^{n} k \cdot \binom{n}{k} = \sum_{k=1}^{n} k \cdot \frac{n}{k} \binom{n-1}{k-1}$$

$$= n \sum_{k=1}^{n} \binom{n-1}{k-1} = n \sum_{l=0}^{n-1} \binom{n-1}{l} = n \cdot 2^{n-1}.$$

$l = k-1$

ALGEBRA.

$$(*) \quad (1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

zróżniczkujmy po x

$$n(1+x)^{n-1} = \sum_{k=1}^{n} \binom{n}{k} k x^{k-1}$$

METODA ANALITYCZNA.

wstawiamy $x \leftarrow 1$:

$$n 2^{n-1} = \sum_{k=1}^{n} k \binom{n}{k}.$$

Zadanie: scałkuj (*).