Counting-sort and Routing in a Single Hop Radio Network

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Model of computation

Radio network

- p stations communicating by radio messages
- single-hop
- synchronized (time is slotted)
- single message in single slot
- single communication channel
- broadcasting/listening to a single message requires unit of energetic cost
- the costs of internal computations is ignored

Energetic cost of the algorithm

The maximal energy dissipated by a single station.

Statement of the sorting problem

- We have p enumerated stations $a_0 \dots a_{p-1}$
- We have n integer keys from the range $[0, 2^m - 1]$
- Each a_i stores s_i keys (with r_i distinct values) (and is destination of d_i keys)
- We want to rearrange the keys that they are distributed among the stations sorted according to their values

We assume that a single message contain either single key or an integer between 0 and n.

Statement of the routing problem

- We have p enumerated stations $a_0 \ldots a_{p-1}$
- Each a_i stores s_i items to r_i distinct stations
- Each a_i is destination of d_i items from q_i other stations
- We want to deliver all items to their destinations

Algorithms for sorting

Singh and Prasanna (2003)

Sorting algorithm based on quick-sort and balanced selection (each station stores single key) with

$$T = \Theta(n \log n) \text{ and } E = \Theta(\log n)$$

Kik (2006)

Sorting based on merging (each station stores $\frac{n}{p}$ keys) with

$$T = (3n + 2p - 2)\log_2 p$$

and

$$E = 8\frac{n}{p}\log_2 p + 2(\log_2 p + 1)\log_2 p$$

Algorithms for routing

Nakano, Olariu and Zomaya (2001)

Routing n packets between p stations

$$T = (2\left\lceil \frac{\log p}{\log n/p} \right\rceil + 1)n + 1 \text{ and } E = (4\left\lceil \frac{\log p}{\log n/p} \right\rceil - 1)\frac{n}{p}$$

Datta and Zomaya (2004)

$$T = 2n + p^2 + p + 2 \text{ and } E = 6\frac{n}{p} + 2p + 8$$

Each station stores $\frac{n}{p}$ and is destination for $\frac{n}{p}$ packets.

Randomized algorithm for routing

Nakano, Olariu and Zomaya (2002)

For every $f \geq 1$ the task of routing n items in p stations can be completed with probability exceeding $1 - 1/f$ with

$$T = n + O(q + \ln f)$$

and

$$E(a_i) = s_i + d_i + O(q_i + r_i \log p + \log f)$$

where $q = \sum_{i=0}^{p-1} q_i$.

Our results

Counting sort

For the single hop and single channel radio network with p stations there exists sorting algorithm for n m-bits integer keys that works with

$$T = mn + n + p \text{ and } E(a_i) = 3mr_i + d_i + s_i + 3$$

Our results

Routing

For the single hop and single channel radio network with p stations there exist routing algorithms with

$$T = r\lceil \log_2 p \rceil + n + r + 3p$$

$$E(a_i) = (3\lceil \log_2 p \rceil + 4)r_i + s_i + d_i + 6$$

where $r = \sum_{i=0}^{p-1} r_i$.

Key procedure - Counting Rank

Main idea

- Compute the ranks of all keys
- Start with ranking which depends only on initial position of the keys
- Refine ranking by considering sequentially bits positions (starting from most significant bit)
- As result we obtain the ranks in sorted sequence

Key procedure - Counting Rank

Algorithm

1. Init procedure – count the total number of keys and ranks all keys by on initial positions (All elements are in single group)
2. For $i \leftarrow m - 1$ down to 0 do:
   Divide each group into two new groups
   group containing keys with 0 on i-th position
   group containing keys with 1 on i-th position
   and ranks elements in these new groups.

Complexity

$T = p + mn$ and $E(a_i) = 3mr_i + 3$

Sorting

1. Counting rank
2. Route by ranks – function of destination of key with rank r is known, for example $dest(r) = \lfloor p \cdot r/p \rfloor$)

Complexity

$$T = mn + p + n \text{ and } E(a_i) = 3mr_i + 3 + d_i + s_i$$

Routing

In the case of routing the keys are numbers of destinations stations from range $[0, p-1]$.
Thus $m = \lceil \log_2 p \rceil$.

1. Counting rank
2. Compute intervals – continuous slots of time where station a_i listen
3. Finish routing

Complexity

$$T = n\lceil \log_2 p \rceil + n + 3p$$

$$E(a_i) = (3\lceil \log_2 p \rceil + 2)r_i + s_i + d_i + 5$$

Routing – acceleration

- During Counting rank each station pretends that has at most one item destined for any receiver
- After Counting rank are computed real ranks

Complexity with acceleration

$$T = r\lceil \log_2 p \rceil + r + n + 3p$$

$$E(a_i) = (3\lceil \log_2 p \rceil + 4)r_i + s_i + d_i + 6$$

$(r \leq \min\{n, p(p-1)\})$

Thank you!