Introduction

Model of computation:
- radio network (set of stations communicating by radio messages)
- single hop (all stations are within the range of each message)
- synchronized (time is divided into slots)
- single channel (in one time slot only one message can be broadcast)
- if station listens then probability of successful reception is \( p \)
  (In reliable network \( p = 1 \).
  In unreliable network \( p < 1 \))
- during each time slot any station can be either:
  - idle (using no energy), or
  - broadcasting (using one unit of energy)

Complexity measures:
- energetic cost – maximum over all stations of the energy used. (Stations are powered by batteries.)
- time – number of time slots used by the computation.

Remark: By “\( L \)” we mean “\( \log n \)”

Ranking

(We assume that \( n = 2^k \)) A sorted sequence of keys: \( b_0, \ldots, b_{n-1} \) permuted by a fixed permutation \( v_3 \) is transmitted periodically. (In time slot \( i \) the key \( b_i \) is broadcast, where \( x = 2^i \mod n \).

Station \( a \) containing key \( b_i \) has to compute the rank of key \( b_i \) in \( b_0, \ldots, b_{n-1} \) (or its approximation).

(Rank of key \( b_i \) in \( b_0, \ldots, b_{n-1} \)) Station \( a \) contains variables \( \minR \) and \( \maxR \) that are the lower and upper bound on the rank, respectively. Initially \( \minR = 0 \) and \( \maxR = n-1 \) can start its computation in arbitrary time slot. In time slot \( i \), the station does:

\[
\begin{align*}
\text{let } s &= 2^i \mod n \quad &\text{if } \minR = \maxR \text{ then stop, otherwise}\;\text{continue}
\end{align*}
\]

Note that the rank of any key \( b_i \) is always in the interval \( [\minR, \maxR] \).

Lemma 1. Let \( k \) be an integer. After \( r \) time slots \( \minR = \maxR \) (i.e. the exact rank is computed) with probability at least \( 1 - (2r)^{-2^k} \).

(Station \( a \) has chances of receiving the direct neighbors of key \( b_i \)).

Lemma 2. The expected value of \( \Delta = \maxR - \minR \) after \( n \) time slots is not greater than \( 2/n - 2 \).

(Broadcasting ordering permutation \( \beta_0 \). Illustrated by the figure to the right. Each \( x \) is connected by vertical dotted line with the node labeled \( \beta_0(x) \).

(Gray level of the tree position and the tree position within the level can be easily read from binary representation of \( x \). Hence, \( \beta_0 \) is “easily computable” function.) The network is reliable and \( v_3 = \beta_0 \) and \( a \) starts in time slot 0 then the energy used by \( a \) is at most \( 1 \).

(If \( \minR = 0 \) and \( \maxR = n-1 \) then the energy used by \( a \) is at most \( n/2 \).)

Theorem 1. If \( v_3 = \beta_0 \) and \( p = 1 \) and station \( a \) starts in arbitrary time slot, then the station \( a \) listens at most \( 1/n + 1/n \) times before it learns its rank.

(Ranking with \( \beta_0 \) in a reliable network)

Theorem 2. For \( 0 < q < 1 \), the procedure \( \mathcal{P} \) sorts any input sequence with probability greater or equal \( 1 - q^{\mathcal{P}} \).

(time / \( n/2 \) to \( n/2 \) + 1)

(mapping)

This follows from Lemma 1 and from the definition of \( \mathcal{P} \).

Theorem 3. For any input, the expected energy used for listening by any single station in \( \mathcal{P} \) is at most \( \mathcal{O}\left(\frac{n}{(1+1/2)(1-1/2)^{2/2}}\right) \).

(If the computation is successful, then all indexes on level \( k \) are computed before the first levelRanking \( \mathcal{P} \) is received.)

This theorem can be used to bound the energy used by the first levelRanking \( \mathcal{P} \).

Bibliographic coordinates of the conference paper:


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