Braid Chain Radio Communication

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Ad hoc Radio Network

model assumptions

Model

radio network	a network consists of nodes communicating via radio channels,
distance bounded communication	two nodes can communicate provided that they are close enough
ad hoc	the location of nodes can be neither planned nor controlled, in particular the nodes may join and leave the network or change their positions
multi-hop	the source and destination nodes are often far away from each other and messages must be processed via intermediate nodes
self-organizing	there is no central control, the network must self-organize itself in a distributed way

Ad hoc Radio Network

model assumptions

Model

position	each node knows its physical position
network discovery	each node knows its neighbors
radio channel(s)	there is a number of radio channels (frequency or time separation)
channel assignment	we assume that each node is assigned a private input channel (due to multi-hop architecture the same physical channel can be reused at different places)

Collision problems

Message collisions

- there is no a priori coordination between nodes
- two neighbors of node A may decide to transmit to A at the same time
- in this case a collision occurs and no message comes through

radio channel assignment

- intended transmission requests unpredictable
- hybrid network with many independent processes run
- $\Rightarrow$ orchestrating radio channel requests impossible in the general case

Anti-Collision mechanism

Cai-Lu-Wang algorithm

assume that a node A at time t wishes to send a message:

1. it chooses $r \in [t, t + \Delta]$ at random
2. it probes the radio channel at time r:
   2.1 if the channel is busy, then A waits until the end of transmission, and starts a new attempt
   2.2 if the channel is not busy, then A starts its own transmission

idea: the node that chooses the smallest r will get access to the channel and start its transmission

Cai-Lu-Wang algorithm

Failure situation

- if A checks the channel status at time r, then it may start blocking the channel at time $r + \delta$ where δ is a hardware related delay, $\delta > 0$ (!)
- it may happen that another node B checks the channel status at time $t \in (r - \delta, r + \delta)$ and based on its status will start the transmission making a collision with A:
  - if $t > r - \delta$, then A does not detect activity of B
  - if $t < r + \delta$, then B does not detect activity of A

consequences

- if $\Delta \gg \delta$, then failure probability is negligible
- big Δ results in higher communication latency, e.g. if only one node attempts to transmit, then there is an additional average delay of $\Delta/2$ per hop

Cai-Lu-Wang algorithm

consequences

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In order to simplify the notation we change the time scale and assume that $\Delta = 1$ from now on, and that the collisions do not occur

Routing problem

Which route to choose from *A* to *B*?

- no global view of the network
- each intermediate node knows only its neighbors and their positions
- other problems like holes in some areas ...

assumption

the network is dense enough so that transmitting the message to a few nodes closer to the destination eventually enables successful delivery

Adversary/Failures

Risks of a single path

- a single dis-functioning node on a path may disrupt communication
- an adversarial/unfair node may disrupt communication on purpose
- a local congestion may delay transmission

conclusion: to be on the safe side it might be better to route a message via alternative paths

Independent paths

The risk of disruption

- a path where on each hop it may be disrupted with a certain probability will eventually be disrupted
- having a few independent paths helps but not radically: the expected live path length is a maximum of two random variables

for details see:
Jacek Cichoń and Marek Klonowski.
On flooding in the presence of random faults.
Fundam. Inform., 123(3):273–287, 2013.

Our Algorithm

a message m is sent from node A to node B:

routing strategy:

- m is routed only via nodes that are in distance at most d from the line AB
- if node C gets m then it forwards it to all such nodes that are closer to the destination
- as conflicts may occur, the nodes follow Cai-Lu-Wang mechanism independently on each connection

d is a parameter that must be set according to nodes' density.

Our Algorithm

Behavior

- ideally, each node should forward m to a few nodes, say k, so that k different intermediate nodes are at each stage
- there are not k independent paths but *collective* processing
- if at some place a node C fails to forward m, then the following nodes get m from other nodes

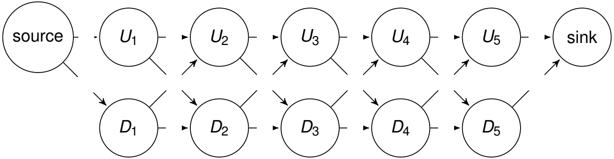
Behavior of the algorithms

Question

what is the message propagation speed? Delays are due to Cai-Lu-Wang mechanism

Braid chain architecture

Idealized case



- there is some number of *layers*
- on each intermediate layer there are exactly 2 nodes
- a nodes forwards a message to both nodes on the next layer

(in reality, there might be layers with more than 2 nodes and layers with exactly one node)

Pseudocode

```
start clock
Loop
    if station $S_i$ receives a message $M$ at time $T$ then
        $T_U := T + \mathrm{random}(0,1)$
        $T_D := T + \mathrm{random}(0,1)$
        while time not later than $\max\{T_U, T_D\}$
            if the current time is $T_U$
                if channel for $U_{i+1}$ is free then
                    transmit message $M$ to $U_{i+1}$
            if the current time is $T_D$
                if channel for $D_{i+1}$ is free then
                    transmit message $M$ to $D_{i+1}$
```

Observed behavior

Question: CLW delay

On each layer we loose some time to to application of Cai-Lu-Wang mechanism.

In this way, how much we loose in total?

2 independent paths

- at each hop the average CLW delay is $\frac{1}{2}$ (or $\frac{\Delta}{2}$ if we forget time rescaling)
- this sum up to CLW delay of $\approx \frac{n}{2}$ for a path of length n on each line
- final total CLW delay is a minimum for delays on both paths (minimum of two random variables of average value $\approx \frac{n}{2}$)
- the variance of these random variables relatively decreases with n, so the final CLW delay $\approx \frac{n}{2}$

braid chain

- a node A gets m from two nodes from the previous layer
- simple case: the nodes at the previous layer receive m at the same time:

  A starts receiving m after time t, where $t = min(t_1, t_2)$, where t_1, t_2 are independent and uniformly distributed in $[0, 1]$

- regular case: the nodes at the previous layer receive m at different times and the time difference is **a random variable of an unknown distribution**
- this has a significant impact on the final CLW delay

Observed behavior

much faster propagation than in case of 2 independent path

Layer delay

definition

- let r_{U_i} be the time when U_i receives M for the first time.
- let r_{D_i} be the time when D_i receives M for the first time.

- $d_i = |r_{U_i} - r_{D_i}|$ is called the layer delay at layer i

Observations

- $d_i \leq 1$
- small d_i speeds up transmission as quickly two nodes attempt to forward a message

Layer delays as a Markov process

Markov process

- layer delay on level $i+1$ is a random variable that depends only on layer delay on level i
- given delay at level i, what is the probability distribution for this variable?
- model:
    - x, y, u, v - independent random variables with the uniform distribution over $[0,1]$
    - we fix $d \in [0,1]$
    - we consider random variables X_d, Y_d, Z_d:

$$X_d = \min(x, d+y), \quad Y_d = \min(u, d+v), \quad Z_d = |X_d - Y_d|$$

Z_d is the delay on the next layer

Determining Z_d

density of X_d, Y_d

$$\Pr(X_d < z) = \begin{cases} z & \text{if } z \in [0, d]\ , \\ (2+d)z - d - z^2 & \text{if } z \in [d, 1]\ . \end{cases}$$

after differentiating we get density of X_d:

$$f_d(z) = \begin{cases} 1 & \text{if } z \in [0, d]\ , \\ 2 + d - 2z & \text{if } z \in [d, 1]\ . \end{cases}$$

Determining Z_d

- Let $U_d = X_d - Y_d$.
- The density of U_d:

$$f_{U,d}(x) = \int f_d(u) f_d(u-x)\, du \,.$$

- the density function k_d of Z_d equals

$$k_d(x) = \begin{cases} 2f_{U,d}(x) & \text{if } x \in [0,1], \\ 0 & \text{otherwise.} \end{cases}$$

Determining Z_d

after tedious computations

$$\begin{array}{ll}
\frac{2}{3}(4-3d+3d^2-d^3-3x+3dx-3d^2x-3x^2+2x^3) & \text{for } d\in(0,\tfrac{1}{2}] \ \wedge\ x\in[0,d] \\
\frac{2}{3}\left(-d^3-3d^2x+3d^2+2x^3-6x+4\right) & \text{for } d\in(0,\tfrac{1}{2}) \ \wedge\ x\in[d,1-d] \\
2+2d-4x-2dx+2x^2 & \text{for } d\in(0,\tfrac{1}{2}) \ \wedge\ x\in[1-d,1] \\
\frac{2}{3}(2-3x+x^3) & \text{for } d=0 \\
\frac{2}{3}(4-3d+3d^2-d^3-3x+3dx-3d^2x-3x^2+2x^3) & \text{for } d(\in\tfrac{1}{2},1) \ \wedge\ x\in[0,1-d] \\
2(1-x) & \text{for } d\in(\tfrac{1}{2},1) \ \wedge\ x\in[1-d,d] \\
2+2d-4x-2dx+2x^2 & \text{for } d\in(\tfrac{1}{2},1) \ \wedge\ x\in[d,1] \\
0 & \text{otherwise}
\end{array}$$

important: on each interval it is a polynomial

Determining Z_d

- based on this explicit formula we can derive any moment of random variable Z_d
- the most convenient way is to use tools of analytical combinatorics

Stationary distribution of the layer delays

- it is easy to see that the Markov process of layer delays is ergodic and therefore there is a stationary distribution μ
- μ satisfies:

  $$\mu(x) = \int_0^1 \mu(t)k(t,x)\,dt$$

  where $k(t,x) = k_t(x)$ is the just computed density function
-

What is the expected value of μ?

Computing expected value of the stationary distribution

$$\begin{aligned}
\mathbf{E}[\mu] &= \int_0^1 x\mu(x)\,dx \\
&= \int_0^1 x\Big(\int_0^1 \mu(t)k(t,x)\,dt\Big)\,dx \\
&= \int_0^1 \mu(t)\Big(\int_0^1 xk(t,x)\,dx\Big)\,dt \\
&= \tfrac{1}{15}\int_0^1 (t^5-5t^3+5t^2+4)\mu(t)\,dt \\
&= \tfrac{4}{15}+\tfrac{1}{15}\int_0^1 (t^5-5t^3+5t^2)\mu(t)\,dt \\
&= \tfrac{4}{15}+\tfrac{1}{15}\int_0^1 (t^5-5t^3+5t^2)\Big(\int_0^1 \mu(s)k(s,t)\,ds\Big)dt \\
&= \tfrac{4}{15}+\tfrac{1}{15}\int_0^1 \mu(s)\Big(\int_0^1 (t^5-5t^3+5t^2)k(s,t)\,dt\Big)\,ds \\
&= \tfrac{4}{15}+\tfrac{1}{15}\tfrac{110}{378}+\tfrac{1}{15}\tfrac{1}{378}\int_0^1 \mu(s)w(s)\,ds
\end{aligned}$$

for polynomial $w(s) = 10s^9 - \ldots + 144s^2$

- **it works as $k(s,t)$ is piecewise polynomial.**
- $0 \leqslant w(s) \leqslant 34$ for $s \in [0,1]$, so the last integral can be estimated,
- ... or one may proceed in exactly the same way in order to get a better precision

Computing expected value of the stationary distribution

Theorem

$$\begin{aligned} \mathbf{E}[\mu] &= 0.286067 + \epsilon, \text{ where } 0 \leq \epsilon \leq 0.006, \\ \mathbf{Var}[\mu] &= 0.126981 + \delta - (0.286067 + \epsilon)^2, \text{ where } 0 \leq \delta \leq 0.0005. \end{aligned}$$

Rapid convergence to stationary distribution

- layer delays converge to stationary distribution - a basic fact from Markov chain theory
- however it does not mean automatically that convergence is fast (and have no impact on short braid chains)
- we prove that the layer delays converge rapidly to the stationary distribution

Coupling based analysis

- we use coupling technique
- the proof is so simple that it looks as a joke

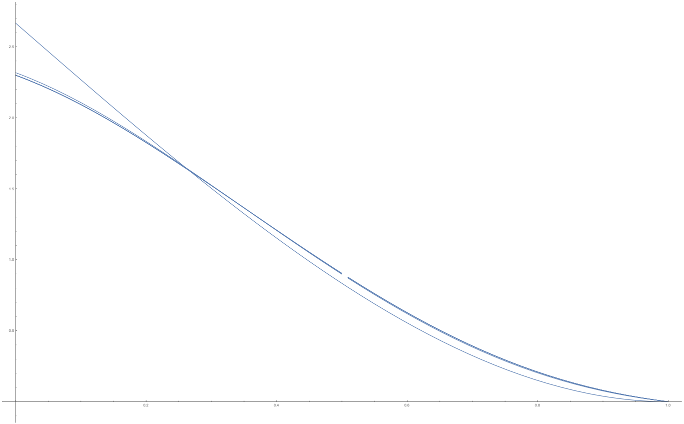
Rapid convergence to stationary distribution

Theorem

For $t > \frac{(-\log \varepsilon + 1)}{2 - \log 3}$ the variation distance between the distributions μ and d_t is at most ε. That is

$$\frac{1}{2} \int_0^1 |\mu(x) - d_t(x)| dx < \varepsilon$$

Rapid convergence to stationary distribution



A plot of 6 consecutive densities for d_t computed numerically according to the formula $k^{(i+1)}(d,x) = \int_0^1 k(z,x)k^{(i)}(d,z)\,dz$

Assumption

we inspect the expected time for a layer delay based on the assumption that the previous layer delay is distributed according to the stationary distribution

Similar computational tricks as before based on the fact that some functions are piecewise polynomials and that integral of the density function yields 1

Theorem

the expected time for one transition is ≈ 0.28.
(the formulas enable computing this constant with an arbitrary precision)

Reusing techniques

choosing transmission time within Cai-Lu-Wang scheme does not need to be uniform.

- other choice: **exponential distribution** (fundamental model for telecommunication)
  – similar analytic results obtained (not included in the paper)
- optimizing Cai-Lu Wang? For some other functions impact on the propagation speed evaluated.

More than 2 nodes on a layer in a braid chain

Increasing the number of nodes on a layer to k decreases the total CLW delay

- for each $k > 2$ we can derive similar formulas as for $k = 2$
  (their complexity grows with k but they are ok for numerical computations)
- **important:** the gain tends to decrease – from propagation speed it does not make sense to increase k more than to 3,4, ...
  the only motivation might be security

Thanks for your attention!

Contact data

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