Extreme Propagation in Ad-hoc Radio Networks

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Computing maximum
General assumptions and motivations

General assumptions

1. wireless communication, multi-hop radio network
2. symmetric links, a single communication channel
3. the network is unstable, the sensors come and go
4. many participants, no external sink supervising the network

Limitations

1. tiny devices, internal memory size
2. limited energy

Motivation

Immediate warning about extraordinary conditions in enviroment

Computing maximum
Model

Network details

Connections can be modeled by a graph G not known in advance:

- if node A sends a message, then all its neighbors in G can hear it
- if a node A gets more than one message at a time, then A cannot understand them

Goal

1. a sensor network, each sensor C_i measuring locally some ξ_i
2. **find** $\max_i \xi_i$ and **propagate it to all nodes**

It is very risky to use an algorithm that is based on the structure of G.

Maximum propagation
Related papers

Baquero, Almeida, Menezes - One round

1: **if** the maximum value c received from neighbors in the previous round exceeds ξ **then**
2: $\xi \leftarrow c$
3: broadcast ξ to all neighbors

Baquero, Almeida, Menezes - Algorithm

Repeat rounds until the values stabilize .

Main features

- the maximum value propagates through the network unaffected by other values
- the time needed is proportional to the maximal distance from the origin of the maximum to the other nodes

Maximum propagation- towards a realistic scenario

analyzed by Cichoń, Lemiesz, Zawada, ADHOC NOW’2012

algorithm executed by a node, round i

$t \leftarrow \mathrm{Random}(i\Delta, (i+1)\Delta)$
if the maximum value c received from neighbors in the previous round exceeds ξ **then**
    $\xi \leftarrow c$
if time t elapsed **then**
    broadcast ξ to all neighbors
    $t \leftarrow \infty$

- a round takes time Δ, a transmission time neglected
- a sender chooses the starting time of a transmission at random from the interval $(i\Delta, (i+1)\Delta - 1)$
- **if the transmission intervals chosen by a node does not intersect with the intervals chosen by other senders, then the message comes through.**

Detailed results and problems

Expected number of messages sent by a node

line graph $E[MC_{L_n}(a)] = 1 + \sum_{k=1}^{a-1} \frac{2}{2k+1} + \sum_{k=2a}^{n} \frac{1}{k}$

circle $E[MC_{C_n}(a)] = 1 + \sum_{k=1}^{a-1} \frac{2}{2k+1}$

grid in the middle:
$E[MC_{G_{n^2}}(\frac{n+1}{2}, \frac{n+1}{2})] = \mathrm{H}_{n^2} - 1.415467 + O(\frac{1}{n})$
in a corner:
$E[MC_{G_{n^2}}(1,1)] = \mathrm{H}_{n^2} - .7296 + O(\frac{1}{n})$

Problems

1. clocks may be not fully synchronized
   this is a dynamic network!
2. propagation delays
   cannot be excluded
3. do we really need rounds?!

Modified algorithm

Code for a single node for a round-less solution

$\xi \leftarrow x_i$
$t \leftarrow \mathrm{Random}(0, \Delta)$
loop
    wait until time t **or** message received
    if message received at time t' and the value c received is $> \xi$ **then**
        $\xi \leftarrow c$
        $t \leftarrow \mathrm{Random}(t', t' + \Delta)$
    else if time t elapsed **then**
        broadcast ξ to all neighbors
        $t \leftarrow \infty$

Modified algorithm
Most important changes

1. no round concept
2. each nodes has local view on rounds
3. if a new maximum arrives then the new round starts

Questions:

1. congestion problem
2. influence on total time to stabilize
3. influence on the number of messages

small quality loss would be tolerable, as there are no guarding times between rounds
sometimes we get an improvement

Time to stabilize

Experiment

Graph A–B–C, maximum has to be transmitted from A to C.

Old algorithm - not tuned

- expected time 1.5Δ: $\Delta + 0.5\Delta$

Asynchronous algorithm

- expected time Δ: $\Delta/2$ to B and $\Delta/2$ to C from B

Old algorithm tuned

- expected time $\frac{13}{12}\Delta$:

$$\int_0^1 \left((1-p)\cdot(p+0.5(1-p)\Delta)+p\cdot 1.5\Delta\right)\,dp$$

Message complexity

Congestion

The probability of collision is negligible for Δ chosen in the same manner as for previous algorithms.

Stochastic process

- algorithm defines a highly complex stochastic process.
- hard mathematical problem, difficult analysis.

Special cases, experiments

- nevertheless, experiments and partial results provide evidence that it is more efficient

Some types of graphs

Star graph

Let n be the number of nodes and M be a random variable denoting the number of retransmissions of the central node, then $\mathbf{E}[M] \leq \frac{3}{2} - \frac{1}{n}$.

Complete graph

For complete graph K_n, the expected total number of messages sent is $\mathrm{H}_n \approx \ln n + 0.577$

If nodes do not know that it is a complete graph then the situation for each node is as for the star graph.

Linear graph

For linear graph L_n, the expected total number of messages sent is $\mathrm{H}_n \approx 2 \ln n + 1.154$

Linear Graph

Network Size	Number Tests	Avg. Msg.Sent	Max.Avg. Msg.Sent	Max. Msg.Sent
20	1000	3.05	4.7	8
100	100	4.75	5.82	14
250	100	5.62	6.82	15
500	100	6.42	7.93	17
1000	60	7.09	8.60	18
2000	20	7.78	9.13	21

Tablica : Simulations for linear graph L_n

Burst values

Linear graph, values sorted

Rule

Bigger value travelling behind a smaller value can sometimes catch up the smaller value and *kill it*(impossible in synchronous model)

Killing neighbors immediately, linear graph(sorted)

- at least $n/4$ messages expected to be killed already in the time period $[0, 1]$. (In reality, more killed!)

Killing slightly later

- there is a chance to catch up a bit later
- until the values in the graph are at a large distance

Future work

Main conclusion

Algorithm is effective and easy to implement in real scenario.

Plans

- compute the expected value of the maximum number of messages sent by a single node for different kind of graphs
- more general results
- compute the expected value of the maximal number of steps of the presented protocol

Thank you!

Contact data

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