

# Step-out Ring Signatures

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# Digital Signatures

## Procedures :

- key setup:
  - private key - for creating a signature
  - public key - for verifying a signature
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Verifying signature  $s$  of  $M$  :

Bob takes the public key  $p_{Alice}$  and checks if

$$\text{test}(s, M, p_{Alice}) = \text{true}$$



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- 2  $test(s, M, p_{Alice}) = false$ , if  $M$  has been changed after creating signature  $s$ ,
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- 2  $\text{test}(s, M, p_{Alice}) = \text{false}$ , if  $M$  has been changed after creating signature  $s$ ,
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## So

**if  $\text{test}(s, M, p_{Alice}) = \text{true}$ ,**  
**then only the holder of  $k_{Alice}$  (i.e. Alice) could produce  $s$  for message  $M$ .**



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- 4 it is infeasible to detect which ring member created a signature.
- 5 **the signer is perfectly hidden in the ring.**
- 6 **one cannot prevent being a member of a ring.**



# Malicious Application of Ring Signatures

## Leaking information

A member of a group (i.e. a parliament commission) can leak a secret information to the press.

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As soon as public keys (e.g. RSA keys) of the commission members are published, nothing can prevent this scenario!



# Step-out Signatures – Target Applications

## Electronic auction

### Requirements:

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## Ring signatures?

- 1 a ring signature authentication and anonymity,
- 2 however, there is no way to force the winner to reveal himself!

a useless solution ...

# Step-out Signatures

## Properties

**Anonymity:** ring type signature: identity of the signer(s) is hidden among identities of non-signers in a ring.

**Confession procedure:** the real signer can prove that he has participated in signature creation.

**Step-out procedure:** a non-signer can prove that he has not participated in signature creation.



# Step-out Signatures

## Properties for auction protocol

**Strong anonymity:** necessary for fairness of e-auctions.

**Confession procedure:** the real signer of the winning bid can reveal himself against the auction.

**Step-out procedure:** a non-signer of the highest bid can step out during the auction and withdraw the deposit.



# Discrete Logarithm

## DL hardness

we use a cyclic group  $G$  such that

- computing  $g^x$  is easy for each  $g, x$
- given a random  $y$ , it is infeasible to find  $x$  such that  $y = g^x$ .

## Secret keys

Each user  $U$  has its private key  $x_U$  selected at random  
the corresponding public key is  $y_U = g^{x_U}$ , where  $g$  is a fixed generator of  $G$ .



# Non-interactive Zero Knowledge Proofs

## Proof of knowledge of discrete logarithm

A signer with a private key  $x$  and a public key  $y$  can prove that  
he knows discrete logarithm of  $y$  (i.e.  $x$ )  
in a non-interactive protocol that reveals no information on  $x$ .



# Non-interactive Zero Knowledge Proofs

## Proof of equality of discrete logarithms

A signer with a private key  $x$  and a public key  $y$  can prove for  $y_1 = g_1^x$  that

$$\log_g y = \log_{g_1} y_1$$

in a non-interactive protocol that reveals no information on  $x$ .

## Proof of equality of discrete logarithms, 1 out of $n$

- Given  $(y_1, g_1), \dots, (y_n, g_n)$  prove that

$$\log_g y = \log_{g_i} y_i \text{ for some unrevealed } i$$

- the proof can be uniquely bound to a message  $m$



# Signature Creation

## Setup

- generators  $g$  and  $\hat{g}$ ,
- ring members with public keys  $y_1, \dots, y_k$
- the signer holds  $y_j$  and the private key  $x_j$

## Signature

proof of equality of discrete logarithms depending on  $m$  and created with  $x_j$



# Signature Creation

## Details

- 1  $r_1, \dots, r_n$  chosen at random,
- 2  $w_j \leftarrow g^{r_j}$ , for  $i = 1, \dots, n$ <sup>a</sup>
- 3  $\hat{w} \leftarrow \hat{g}^{r_j}$ ,  $\hat{y} \leftarrow \hat{g}^{x_j}$ ,
- 4 the signature is a non-interactive zero knowledge proof (depending on  $m$ ) that

$$\log_{\hat{g}} \hat{y} \hat{w}$$

equals one of the logarithms

$$\log_g(y_1 w_1), \dots, \log_g(y_n w_n)$$



# Signature Verification

## Idea

Simply checking the non-interactive zero knowledge proof provided by the signature



# Revealing the Signer

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- 2 the same parameters  $\hat{w}\hat{y}$  and  $w_1, \dots, w_n$  are used in both proofs— this enforces that  $\log_{\hat{g}}(\hat{w}\hat{y})$  occurs on both lists:

$$\log_g(y_1 w_1), \log_g(y_2 w_2) \dots, \log_g(y_n w_n)$$

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so it must be  $\log_g(y_1 w_1)$  as it is the only common element.

- 3 recall that the element of the same discrete logarithm has been created by the signer!

# Step out

## Idea of stepping out of the ring

- 1 a signature  $s$  contains  $\hat{y}\hat{w}$  and  $w_1, \dots, w_n$ ,
- 2 a non-signer  $A$  provides two step-out signatures for the message *"I have not signed  $m$ "*,
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- 4 the same strings  $w_i$  are used, ...
- 5 but with  $\hat{y}'\hat{w}'$  instead of  $\hat{y}\hat{w}$ .
- 6 however,  $\hat{y}\hat{w}$  is uniquely determined by  $w_i$ , if  $y_i$  corresponds to the signer!

**So the signer cannot create these additional signatures.**



# Thank you for your attention

