## Fair Mutual Authentication

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## Mutual authentication

## Goal of mutual authentication

- Alice and Bob communicate online
- Alice wants to know that she really talks with Bob
- Bob wants to know that he really talks with Alice


## Protocol example

## Authentication via a shared key $K$

1 Bob chooses random $N_{B}$ and sends it to Alice,
2 Alice chooses random $N_{A}$ and sends it and $P_{A}=\operatorname{Hash}\left(K, N_{A} \| N_{B}\right.$, "Alice, Bob") to Bob,

3 Bob computes $P_{A}^{\prime}=\operatorname{Hash}\left(K, N_{A} \| N_{B}\right.$, "Alice", "Bob") and aborts if $P_{A}^{\prime} \neq P_{A}$,

4 Bob returns $P_{B}=\operatorname{Hash}\left(K, N_{A} \| N_{B}\right.$, "Bob", "Alice") to Alice,
5 Alice computes $P_{B}^{\prime}=\operatorname{Hash}\left(K, N_{A} \| N_{B}\right.$, "Bob", "Alice") aborts if $P_{B}^{\prime} \neq P_{B}$,

6 Alice, Bob: accept if not aborted

## Tracing Problem

- at step 3 Bob learns that he is talking with Alice
- until step 5 Alice learns nothing


## GDPR and privacy-by-design

Mutual authentication protocol turns to be an effective tracing tool.

The location of a physical person is under protection.

No-tracing possible - by design!

## Markov Fair Mutual Authentication

## Idea

- Alice and Bob exchange the authenticating information bit-by-bit
- some bits sent are false at random moments

■ ... nevertheless no partner has a substantial information advantage at any moment

## False bits versus cryptanalysis

!!! an observer has no idea which bits are correct
$\Longrightarrow$ like for Learning Parity With Errors: cryptanalysis becomes substantially harder

## Details

Let $P_{A}=a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ and $P_{B}=b_{1}, b_{2}, b_{3}, \ldots, b_{n}$, $p \in[0,1]$ - a probability parameter

## Round $i$

let $\Delta_{i}$ be the difference between the number of erroneous bits sent by Alice and Bob.

■ if $\Delta_{i}=-1$, then Alice sends $a_{i}$,
$\square$ if $\Delta_{i}=0$ or $\Delta_{i}=1$, then Alice sends $a_{i}$ with probability $p$ and $\neg a_{i}$ with probability $1-p$,

- if $\Delta_{i}>1$, then Alice enters the failure state and from now on sends random bits.


## Features

GDPR: no tracing, $\approx$ same amount of personal personal data bits exchanged in each direction regardless of protocol run
lightweight: due to erroneous bits, relatively weak hash function can be used as well as small number of bits exchanged. IoT friendly!

## Markov chain

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## Differences as a Markov chain

Stochastic process $\left\{\Delta_{i}\right\}_{i}$ examined
$\Delta_{i}=$ the difference between the numbers of correct authentication bits sent by Bob and Alice up to round $i$

It is a Markov chain with states $\mathbf{- 1 , 0 , 1}$ and a failure state F .

## Fair Execution



- optimal choice for parameter $p$ is $\frac{2}{3}$
- process very quickly converges to the stationary distribution: $\pi=\left(\frac{2}{7}, \frac{3}{7}, \frac{2}{7}, 0\right)$
- expected fraction of incorrect bits $\approx \frac{1}{4}$
- incorrect bits well distributed


## Execution with a Party Impersonating Bob



■ The most critical moment from the point of information leakage is a visit in the state -1 . In this case, Alice must send the correct bit.

- the number of visits of the state -1 during a protocol execution is a random variable $Z$

■ it should be small!

- for for $p=\frac{2}{3}$ :

$$
E[Z] \frac{3}{2}, \quad \operatorname{Var}[Z]=\frac{27}{4}
$$

## Thank you for your attention!

## Acknowledgments

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