Hierarchical Ring Signatures

Łukasz Krzywiecki, Mirosław Kutyłowski,
Anna Lauks-Dutka

Institute of Mathematics and Computer Science
Wrocław University of Technology

WEWoRC 2009
July 7-9, Graz University of Technology

Outline

1. Ring Signatures
   - Concept
   - Problem
2. Hierarchical Ring Signatures
   - Idea
   - Building Blocks
   - Scheme Description
   - Signatures Graph

Outline

1. Ring Signatures
   - Concept
   - Problem
2. Hierarchical Ring Signatures
   - Idea
   - Building Blocks
   - Scheme Description
   - Signatures Graph

The Concept of Ring Signatures - [1]

Ring signature = digital signature used to sign a document in anonymous way

Basic Properties

1. Signer uses his private key and public keys of some arbitrary group of people
2. Identity of the signer is hidden within this group (called a ring)
3. One cannot prevent being involved into a ring

[1] R.L. Rivest, A. Shamir, Y. Tauman: "How to Leak a Secret"

The Concept of Ring Signatures - [1]

Ring signature = digital signature used to sign a document in anonymous way

Basic Properties

1. Signer uses his private key and public keys of some arbitrary group of people
2. Identity of the signer is hidden within this group (called a **ring**)
3. One cannot prevent being involved into a ring

[1] R.L. Rivest, A. Shamir, Y. Tauman: ”How to Leak a Secret”

The Concept of Ring Signatures - [1]

Ring signature = digital signature used to sign a document in anonymous way

Basic Properties

1. Signer uses his private key and public keys of some arbitrary group of people
2. Identity of the signer is hidden within this group (called a **ring**)
3. One cannot prevent being involved into a ring

[1] R.L. Rivest, A. Shamir, Y. Tauman: ”How to Leak a Secret”

The Concept of Ring Signatures - [1]

Ring signature = digital signature used to sign a document in anonymous way

Basic Properties

1. Signer uses his private key and public keys of some arbitrary group of people
2. Identity of the signer is hidden within this group (called a **ring**)
3. One cannot prevent being involved into a ring

[1] R.L. Rivest, A. Shamir, Y. Tauman: ”How to Leak a Secret”

The Drawback of Ring Signatures

The public keys of **all** ring members are necessary for verification

Consequences

- the signature size is proportional to the ring size
- higher anonymity level = longer signatures

The Drawback of Ring Signatures

The public keys of **all** ring members are necessary for verification

Consequences

- the signature size is proportional to the ring size
- higher anonymity level = longer signatures

Previous Solution

Observations from [2]

- in practical situations ring does not change for a long period of time
- rings can have implicit short descriptions e.g.:

  ”the ring of public keys of all members of the President’s Cabinet”

The signature size **does not** have to be linear in the size of the ring

[2] Y. Dodis, A. Kiayias, A. Nicolosi, V. Shoup: ”Anonymous Identification in Ad-hoc Groups”

Previous Solution

Signature Scheme from [2]

- based on one-way accumulators
- uses group secret and public keys
- produces constant-size ring rignature

[2] Y. Dodis, A. Kiayias, A. Nicolosi, V. Shoup: ”Anonymous Identification in Ad-hoc Groups”

Outline

1. Ring Signatures
   - Concept
   - Problem
2. Hierarchical Ring Signatures
   - Idea
   - Building Blocks
   - Scheme Description
   - Signatures Graph

The Core Idea of Proposed Solution

Hierarchical Ring Signatures

1. Reuse the information about the previously created rings to get shorter signatures
2. Form a hierarchical structure – signatures created on a particular level utilizes anonymity sets from lower levels

Anonymity set grows exponentially with the level number

The Core Idea of Proposed Solution

Hierarchical Ring Signatures

1. Reuse the information about the previously created rings to get shorter signatures
2. Form a hierarchical structure – signatures created on a particular level utilizes anonymity sets from lower levels

Anonymity set grows exponentially with the level number

Building Blocks of The Construction

Non-Interactive Zero Knowledge Proof of knowledge and equality 1 out of *n* discrete logarithms

Given $(y_1, g_1), \ldots, (y_n, g_n)$ and (y, g) prove that

$$\log_g y = \log_{g_i} y_i \text{ for some unrevealed } i$$

Notation: $\mathrm{NIZKP}(g, y, \{(g_1, y_1), \ldots, (g_n, y_n)\})$

Standard Digital Signature Scheme

$\mathrm{SIG}(g^x, M)$ - signature of the message M.

Assumption: scheme with secret and public keys in the form of (x, g^x)

Hash function

$\mathcal{H} : \{0, 1\}^* \to \langle g \rangle$

Building Blocks of The Construction

Non-Interactive Zero Knowledge Proof of knowledge and equality 1 out of n discrete logarithms

Given $(y_1, g_1), \dots, (y_n, g_n)$ and (y, g) prove that

$$\log_g y = \log_{g_i} y_i \text{ for some unrevealed } i$$

Notation: $\mathrm{NIZKP}(g, y, \{(g_1, y_1), \dots, (g_n, y_n)\})$

Standard Digital Signature Scheme

$\mathrm{SIG}(g^x, M)$ - signature of the message M.

Assumption: scheme with secret and public keys in the form of (x, g^x)

Hash function

$\mathcal{H} : \{0, 1\}^* \to \langle g \rangle$

Building Blocks of The Construction

Non-Interactive Zero Knowledge Proof of knowledge and equality 1 out of n discrete logarithms

Given $(y_1, g_1), \ldots, (y_n, g_n)$ and (y, g) prove that

$$\log_g y = \log_{g_i} y_i \text{ for some unrevealed } i$$

Notation: $\mathrm{NIZKP}(g, y, \{(g_1, y_1), \ldots, (g_n, y_n)\})$

Standard Digital Signature Scheme

$\mathrm{SIG}(g^x, M)$ - signature of the message M.

Assumption: scheme with secret and public keys in the form of (x, g^x)

Hash function

$\mathcal{H} : \{0, 1\}^* \to \langle g \rangle$

Scheme Description

Assumptions

- there is a PKI for registering public keys of the users
- $(x_u, y_u = g^{x_u})$ - the private and public key of user u
- there is a bulletin board (BB) where all hierarchical signatures can be published

Scheme Description

Signature Creation at The Base Level

- $A = (y_1, y_2, \ldots, y_j, \ldots, y_n)$ - ring
- j - the signer
- g_A - generator obtained from $\mathcal{H}$

$$\begin{aligned} \mathrm{SHRS}_A \quad := \quad & \mathrm{NIZKP}(g_A, g_A^{x_j}, \{(g, y_1), \ldots, (g, y_n)\}) \; \| \\ & \| \; \mathrm{SIG}(g_A^{x_j}, M_A) \end{aligned}$$

Signature size at the base level is proportional to the cardinality of the ring

Scheme Description

Signature Creation at The Base Level

- $A = (y_1, y_2, \ldots, y_j, \ldots, y_n)$ - ring
- j - the signer
- g_A - generator obtained from $\mathcal{H}$

$$\begin{aligned}\mathrm{SHRS}_A \quad := \quad & \mathrm{NIZKP}(g_A, g_A^{x_j}, \{(g, y_1), \ldots, (g, y_n)\}) \mid\mid \\ & \mid\mid \mathrm{SIG}(g_A^{x_j}, M_A)\end{aligned}$$

Signature size at the base level is proportional to the cardinality of the ring

Scheme Description

Signature Creation at The Next Levels

- g_C - generator obtained from $\mathcal{H}$
- SHRS_A - hierarchical ring signature created by j
- SHRS_B - hierarchical ring signature created by $i \neq j$

$$\begin{aligned} \mathrm{SHRS}_C \quad := \quad & \mathrm{NIZKP}(g_C, g_C^{x_j}, \{(g_A, g_A^{x_j}), (g_B, g_B^{x_i}\})) \, || \\ & || \, \mathrm{SIG}(g_C^{x_j}, M_C) \end{aligned}$$

Signature size at the next levels is **much lower** then the cardinality of the ring!

Scheme Description

Signature Creation at The Next Levels

- g_C - generator obtained from $\mathcal{H}$
- SHRS_A - hierarchical ring signature created by j
- SHRS_B - hierarchical ring signature created by $i \neq j$

$$\begin{aligned} \mathrm{SHRS}_C \quad := \quad & \mathrm{NIZKP}(g_C, g_C^{x_j}, \{(g_A, g_A^{x_j}), (g_B, g_B^{x_i}\})) \mid\mid \\ & \mid\mid \mathrm{SIG}(g_C^{x_j}, M_C) \end{aligned}$$

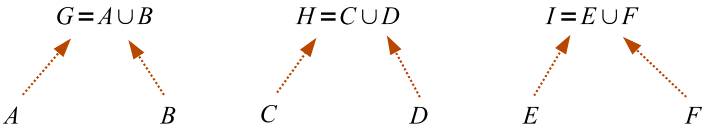
Signature size at the next levels is **much lower** then the cardinality of the ring!

Creating New Signatures

Anonymity Sets at The Base Level

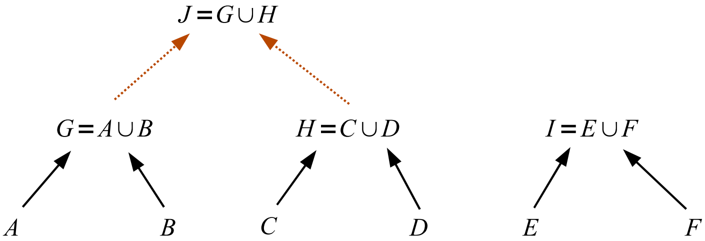
Creating New Signatures

Anonymity Sets at The Second Level



Creating New Signatures

Anonymity Sets at The Third Level



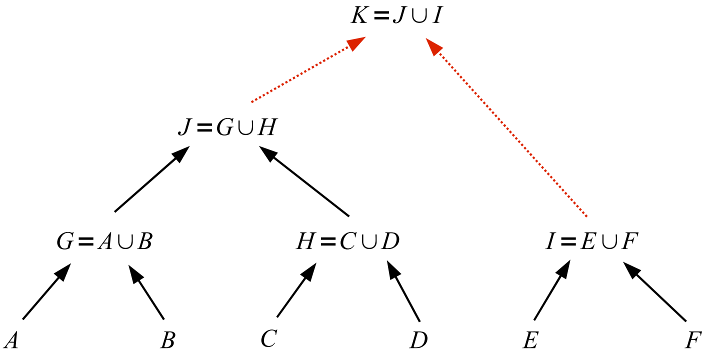
$J = G \cup H$

$G = A \cup B$ $H = C \cup D$ $I = E \cup F$

A B C D E F

Creating New Signatures

Anonymity Sets at The Next Levels



$K = J \cup I$

$J = G \cup H$

$G = A \cup B$

$H = C \cup D$

$I = E \cup F$

A

B

C

D

E

F

Signature Verification

Phases

1. check if SIG$(g_C^{x_j}, M_C)$ verifies correctly with the verification key $g_C^{x_j}$
2. check NIZKP$(g_C, g_C^{x_j}, \{(g_A, g_A^{x_j}), (g_B, g_B^{x_i}\}))$

if OK

- M_C was signed by a user whose private key is hidden in the exponent of $g_C^{x_j}$
- the exponent hidden in $g_C^{x_j}$ is equal to one of the exponents hidden in the elements of the ring A or B

Signature Verification

Phases

1. check if $\mathrm{SIG}(g_C^{x_j}, M_C)$ verifies correctly with the verification key $g_C^{x_j}$
2. check $\mathrm{NIZKP}(g_C, g_C^{x_j}, \{(g_A, g_A^{x_j}), (g_B, g_B^{x_i}\}))$

if OK

- M_C was signed by a user whose private key is hidden in the exponent of $g_C^{x_j}$
- the exponent hidden in $g_C^{x_j}$ is equal to one of the exponents hidden in the elements of the ring A or B

Thank you for your attention!