Mobile Mixing

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How long it takes so that traffic analysis does not provide any substantial information?

Example

single message

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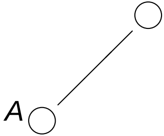

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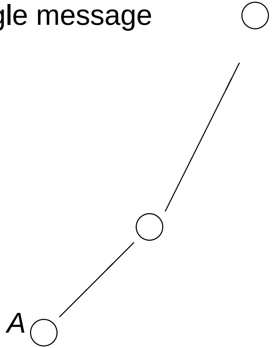

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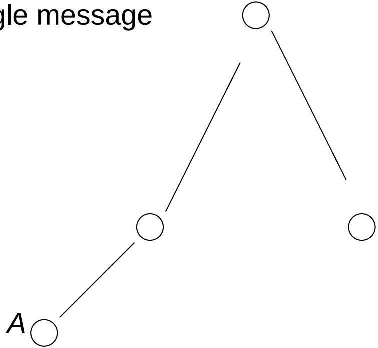

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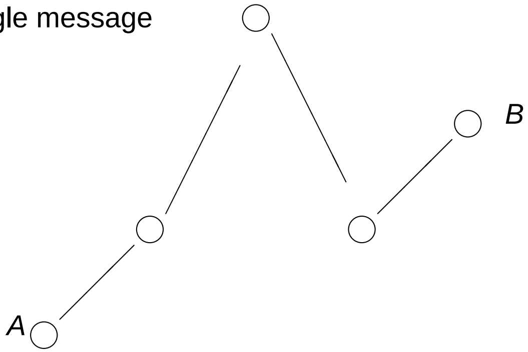

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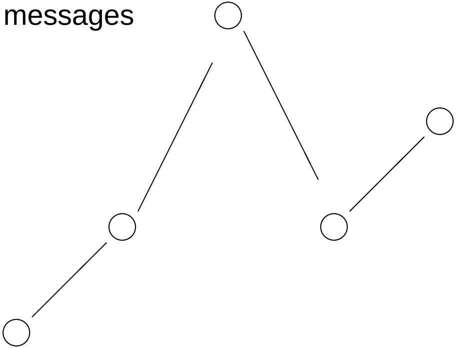
many messages



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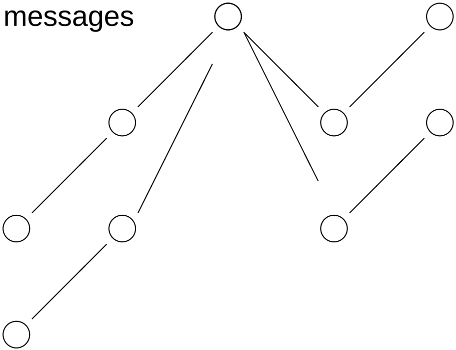

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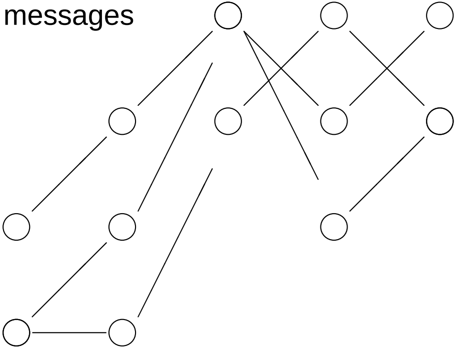

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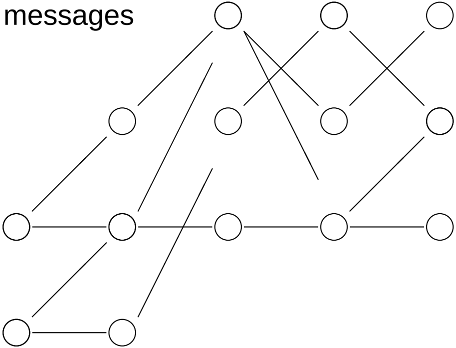

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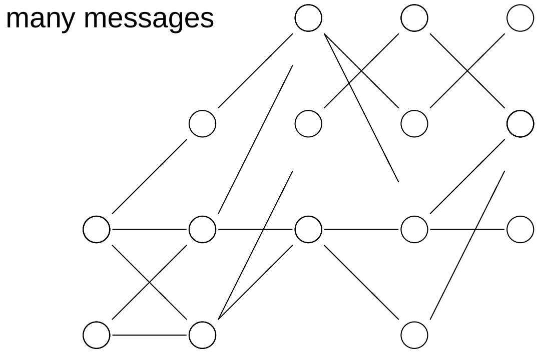

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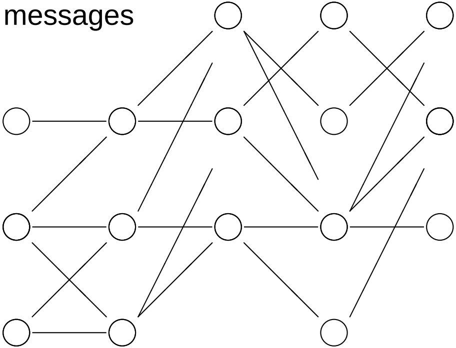

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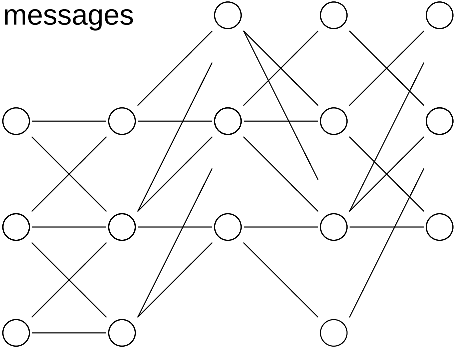

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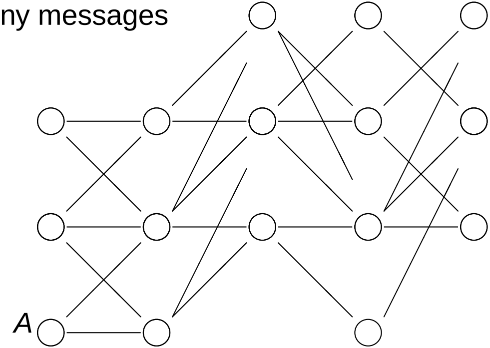

destination of the message starting at *A*?

State of the art

1. intuition: *after a sufficiently long time the origins are hidden*
2. no proofs, case-by-case proof or artificial assumptions
3. no reasonable bounds

Related problems - mixes and onions

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Adversary model

Different adversary models:

- whole traffic can be observed (strong model)
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Related problems – results on mixes

- many attacks and countermeasures for a dynamic adversary
- no security proofs except: a cascade of pairs of mixes, passive adversary – processing through $O(1)$ pairs is enough,
  Gomułkiewicz, Klonowski, Kutyłowski (ESORICS'2003)

Related problems – results on onions

- many attacks and countermeasures for a dynamic adversary
- strong adversary:
  - Rackoff, Simon (ACM STOC'93): polylogarithmic time (degree 11), special assumption: at stage i the messages stay inside groups of cardinality 2^i
  - Czumaj, Kanarek, Kutyłowski, Loryś (ACM SODA'99): under the same assumptions - time $O(\log^2 n)$
- passive adversary:
  - Berman, Fiat, Ta-Shma (FC'2004) – $O(\log^4 n)$ steps for n messages,
  - Gomułkiewicz, Klonowski, Kutyłowski (ISC'2004) – $O(\log n)$ steps

Our model

- **A message cannot be sent to any other node in a network in one step.**
- It can be sent only to the neighbor nodes.

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Problem: Anyone in range can contact an RFID-tag. Even if they cannot write any information, they can trace the tag!

Re-encryption of the RFID-tags does not automatically protect against traffic analysis.

Application – Mobile Agents

Mobile agents:

- program and data migrating through a network
- used for diverse perpuses: processing and search of information in a distributed information system
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Goal: routes of the agents should remain hidden.

Application – Distributed Timestamping

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- A time-stamping requests encoded in an onion.
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Security problem: if traffic analysis suceeds, then an adversary can block timestamping requests of particular users and observe who is issuing timestamps

Process description – adversary view - On hobbits and the rings

The following process well describes our scenario:

- There are m *hobbits*.
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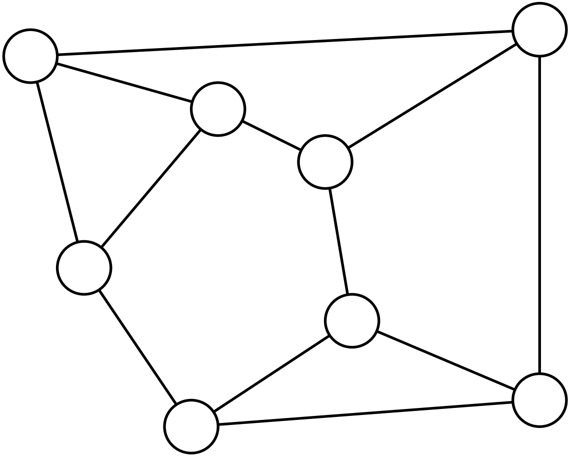
- The hobbits perform (independently) a random walk on a graph G.
- If some hobbits meet in a node they exchange their rings at random.
- An adversary observes the movements of the hobbits (but not how they exchange the rings) and tries to locate the magic ring(s).

Hobbits



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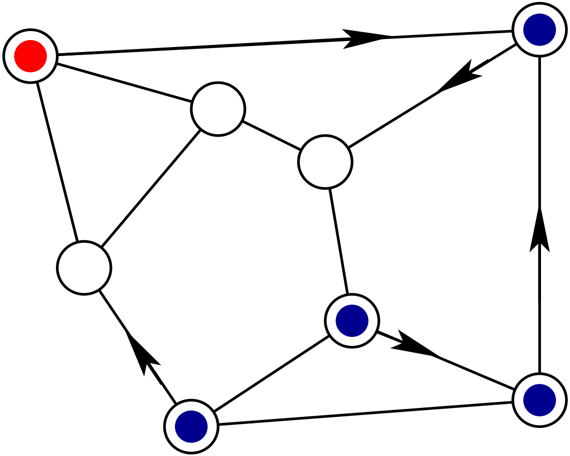

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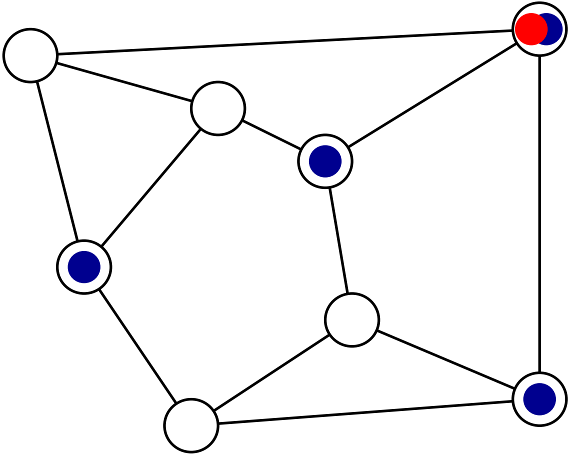

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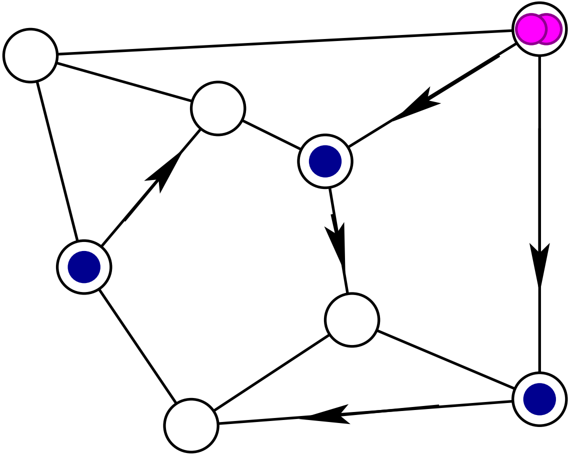

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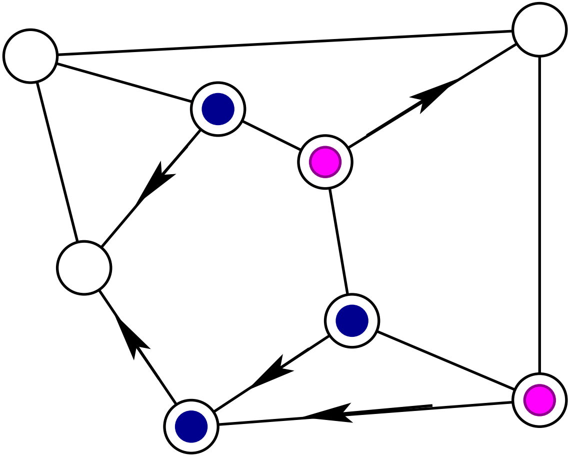

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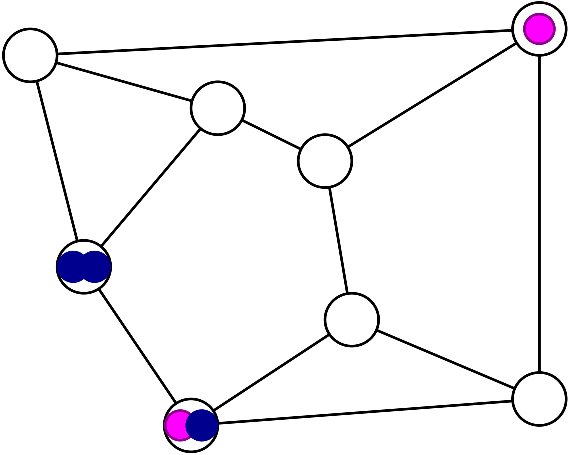

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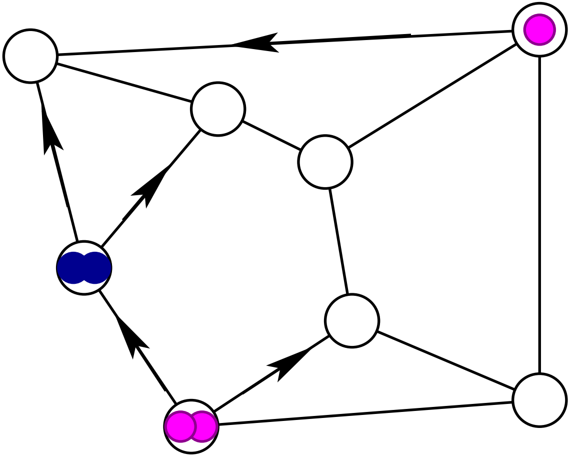

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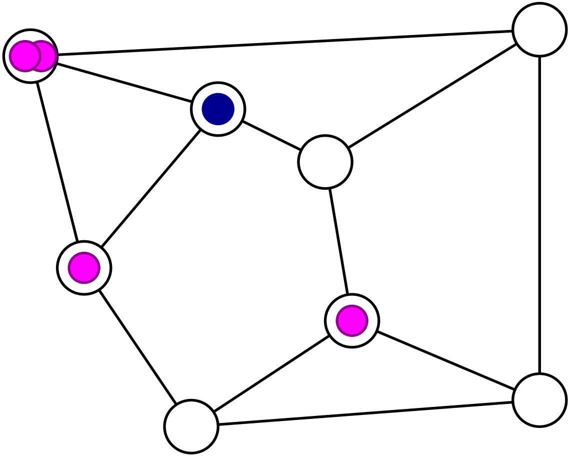

Main problem

How many steps of the hobbits are necessary until the adversary has no substantial advantage from observing the movements of the hobbits?

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- probability distribution **conditioned on the traffic information** should be almost uniform
- **variation distance** as a measure of closeness of two probability distributions

Main result

Let:

- G – n-node regular graph with mixing time τ_G
- $m = \Theta(n)$ – hobbits
- $k \leq \frac{1}{2}m$ – magic rings

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Main Theorem

After $\lambda = \Omega(\tau_G^3 \cdot \log^6 n \cdot \log k)$ steps probability distribution of magic rings is close to the uniform distribution with high probability (over traffic information).

Proof outline

- Our problem is much different from locating the hobbits on random positions in graph G.
- “mixing” occurs dynamically - therefore an analysis is becomes harder.

Proof phases

1. solve the problem for one magic ring - *potential functions*
2. generalize to many magic rings - *path coupling*

The main technical contribution is part 1.

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- $\tau_H(P) = \min_t\{d_H(P;t) \leq 0.1\}$
- τ_G – *mixing time* of graph *G* (it describes time after which "a single hobbit is at a random location in *G*"

Key technical result

Key Theorem

Assume that the mixing time of G is $n^{o(1)}$*.*
Then with probability at least $1-\frac{1}{n}$ *over the choices of P:*

$$\tau_H(P) = O(\tau_G^3 \cdot \log^5 n)$$

Potential functions

- $g(P;t) = \sum_{i=1}^{m} \rho^2(P;i,t)$.
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Some properties:

- $g(P;t)$ is a non-increasing function of t .
- It suffices to inspect when $d_H(P;t_0) \leq 0.1$.

Main Lemma

Lemma

Let $d_H(P;t_0) \geq 0.1$ *for a pattern P. Then for certain* τ_1, τ_0*:*

$$E\left[g(P;t_0) - g(P;t_0+\tau_1+1)\right] \geq \Omega(g(P;t_0)/(\tau_G^2 \log^3 n))$$

Corollary:

By definition $g \geq \frac{1}{7}m$*, so finally* $d_H(P;t_0) < 0.1$*)*

Proof rationale:

if $d_H(P;t_0) \geq 0.1$, then there are many hobbits with high (low) probability of posessing the magic ring. When such hobbits meet, then probabilities get equal and the potential decreases substantially.

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- still unclear what is the **real** relationship between the convergence rate considered and the mixing time
- what about special classes of graphs occuring in practice