

# Security and Cryptography 2021

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## X. CLONE DETECTION and AVOIDANCE

**problem:** a hardware token executing cryptographic protocol can be cloned once the attacker gets access to the internal state of the token with all secrets

## Strategies

- no secrets in full control of one party/device (e.g.: distributed generation of keys)
- making clones useless (rapid changes and synchronization)
- immediate detection of active clones

## Distributed key generation

- split responsibility for the key quality, at least 2 parties involved
- result:
  - i one party learns the key
  - ii 2 parties share a key, but nobody has the entire key

## Easy case – DL based systems

DH based procedure:

1. device  $A$  sends  $X_0 = g^{x_0}$  to device  $B$
2. device  $B$  sends  $X_1 = g^{x_1}$  to device  $A$
3.  $A$  responds with  $x_0$  (maybe encrypted with  $K = \text{Hash}(X_1^{x_0})$ )
4.  $B$  computes the public key  $K = X_1^{x_0}$  and the private key  $x := x_0 \cdot x_1$
5.  $A$  can check that the resulting key is  $K$  but has no knowledge about  $x$

A version where  $A$  and  $B$  keep key shares, respectively,  $x_0$  and  $x_1$

## Hard case – RSA

necessary to derive 2 prime numbers so that neither  $A$  nor  $B$  knows any of these primes

### trick (from Estonian ID cards)

use 4K-bit numbers that have 4 prime factors instead of 2

backwards comp.

observation: the same algebra as for the original RSA show that if

$$e \cdot d = 1 \pmod{\dots}$$

then

$$(m^d)^e = m \pmod{n}$$

Citizen

card



~~k2~~

Gov



~~k1~~



## Smart ID key generation

1. App generates a 2048-bit RSA key pair with the private key  $(n_1, d_1)$  and public key  $(n_1, e)$
2. App chooses  $d'_1$  at random
3. App computes  $d''_1 = d_1 - d'_1$
4. App encrypts  $d'_1$  with its PIN, stores the ciphertext and deletes its plaintext
5. App deletes plaintext of  $d_1$  (and information leading to factors of  $n_1$ )
6. App sends  $n_1, e, d''_1$  to SecureZone
7. SecureZone generates the 2048-bit RSA key pair with private key  $(n_2, d_2)$  for public key  $(n_2, e)$
8. SecureZone computes  $\alpha, \beta$  so that

$$e \cdot d_1 = 1 \pmod{\text{order} \mathbb{Z}_n^*}$$

$$\beta = \frac{1}{n_2} \pmod{n_1}$$

$$\alpha \cdot n_1 + \beta \cdot n_2 = 1 \pmod{n_1}$$

(Euclidean algorithm for integers, it works as  $n_1$  and  $n_2$  are coprime whp).

9. SecureZone computes the user's public modulus  $n = n_1 \cdot n_2$

public key of a user is  $(n, e)$

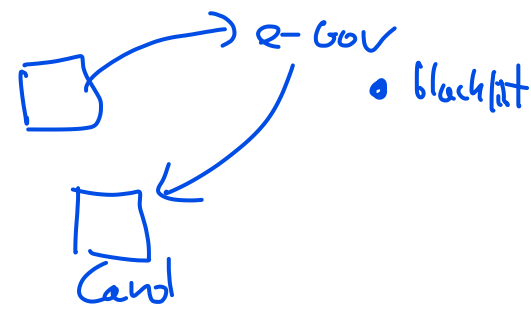
## distributed "RSA" signature generation for $M$

1. App asks for the PIN and decrypts the ciphertext of  $d'_1$
2. App computes  $m$  - encoding of  $M$
3. App computes  $s'_1 := m^{d'_1} \bmod n$  and sends it to Smart-ID Server
4. Smart-ID Server computes  $m$  - encoding of  $M$
5. Smart-ID Server computes  $s''_1 = m^{d''_1} \bmod n_1$
6. Smart-ID Server computes  $s_1 = s'_1 \cdot s''_1 \bmod n$   
(so  $s_1 = m^{d_1} \bmod n_1$ )
7. Smart-ID Server computes  $s_2 = m^{d_2} \bmod n_2$
8. Smart-ID Server computes

$$S := \beta \cdot n_2 \cdot s_1 + \alpha \cdot n_1 \cdot s_2 \bmod n$$

(by ChRT to get  $S$  such that  $S = s_1 \bmod n_1$  and  $S = s_2 \bmod n_2$ )

**output:** signature  $S$



$$s'_1 \cdot s''_1 = m^{d'_1} \cdot m^{d''_1} = m^{d'_1 + d''_1} = m^{d_1} \dots$$

## Verification

as for RSA: checking that  $S^e = m \pmod n$

$$S^e = m \pmod n \quad \text{iff} \quad S^e = m \pmod{n_1} \quad \text{and} \quad S^e = m \pmod{n_2}$$

$$s_1^e = m \pmod{n_1} \quad \text{and} \quad s_2^e = m \pmod{n_2}$$

$$(m^{d_1})^e = m \pmod{n_1} \quad \text{and} \quad (m^{d_2})^e = m \pmod{n_2}$$

$$S^e = m \pmod{n_1} \quad S^e = m \pmod{n_2}$$



## Security concept

$$n = n_1 \cdot n_2$$
$$d_1$$
$$S_1 = m^{d_1} \pmod{n_1}$$
$$S_2 = m^{d_2} \pmod{n_2}$$
$$S$$

in order to create a signature alone:

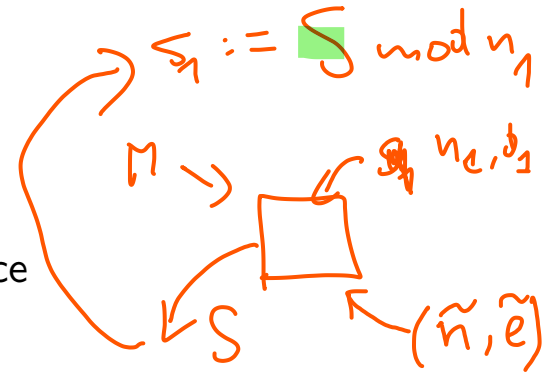
- App would need to create  $m^{d_2} \pmod{n_2}$  – impossible if the original RSA signature is unforgeable
- Smart-ID server would need to create  $m^{d_1} \pmod{n_1}$ . It knows  $n_1$  but the exponent  $d_1''$  is random, so cannot help to forge an RSA signature for modulus  $e$

$$\boxed{\begin{array}{ccc} n_1, e, d_1'' \\ \# & \# & \# \\ \tilde{n}, \tilde{e}, r \end{array}}$$

## Conclusion

distributing private key can work

whereas an adversary can typically clone at most one device



## Clone detection concepts

1. hide invisible characteristics in the device that may be used to fish out clone's signatures post factum
2. discourage to use clones: key compromise in case of clone usage
3. fluctuation of distributed key

9 Dec 1100

## Key fluctuation

works for RSA, EdDSA, Schnorr, ...

fluctuation (example for plain RSA)

– App holds  $d_1$ , Server holds  $d_2$

– signature creation:

i an integer  $\Delta$  is negotiated

ii App updates:  $d_1 := d_1 - \Delta$

iii Server updates:  $d_2 := d_2 + \Delta$

(computations over integers, as the group order is unknown)

$$r = g^k$$

$$s_A = k - e \cdot d$$

$$s_1 = k_1 - e \cdot d_1$$

$$s_2 = k_2 - e \cdot d_2$$

$$d_1 + d_2 = d$$

$$k_1 + k_2 = k$$

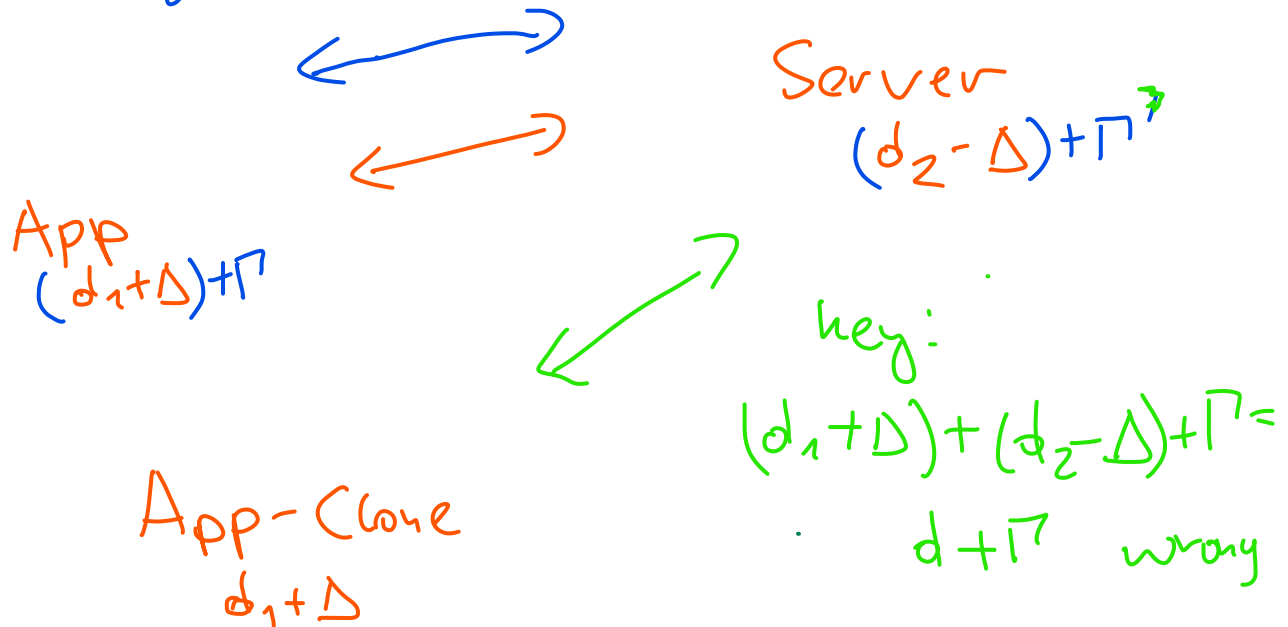
$$d_1 + d_2 = d$$

$$s_1 + s_2$$

## Security concept of key fluctuation

- App and Server must be synchronized
- If App<sub>1</sub> and App<sub>2</sub> are clones, then App<sub>1</sub> de-synchronizes App<sub>2</sub>: if it attempts to sign, then the signature will be invalid and the Server will notice the problem

$$\text{key: } (d_1 + \Delta) + (d_2 - \Delta) - \Delta_2 = d - \Delta_2 \neq d \Rightarrow \text{Sign invalid}$$



## Tokens - example Smart-ID

Clone detection works thanks to the following nonce (original Estonian description):

**one-time password** – created by Smart-ID Core in the end of each operation (incl. initialization) and valid until the completion of next.

**retransmit nonce** – created in the beginning of each operation by Smart-ID App, the same value must be used when Smart-ID App retries messages to Smart-ID Core, related to the same operation.

**freshness token** – created by Smart-ID Core before each submission operation from Smart-ID App to Smart-ID Core. Ensures that state-changing operations get executed in the order client issued them (although some may be missing from between).

$$\text{sign}(M \parallel R')$$

## Linking – microTESLA ...

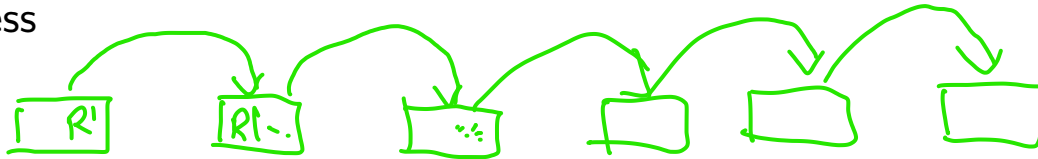
at session  $k$ :

- i.  $A$  chooses  $R$  at random,  $R' := \text{Hash}(R)$  (or an HMAC of  $R$  is MAC key shared)
- ii.  $A$  attaches  $R'$  to the current transmission

at session  $k + 1$ :

- i.  $A$  authenticates himself with  $R$

$\Rightarrow$  if at some moment a clone is created and does not hijack synchronization with the server, then it is useless



## Detection of active clones

idea: clone may emerge, but their holder will never use them without revealing compromise

two examples:

1. failstop signatures
2. commitments

## Failstop signatures

Domain Parameters and Keys:

- $G_q$  – a group of a prime order  $q$  such that DLP is hard in  $G_q$
- $g, h \in G_q$  be such that nobody should know  $\log_g h$
- one-time secret  $SK = (x_1, x_2, y_1, y_2)$
- one-time public key  $PK = (g^{x_1} h^{x_2}, g^{y_1} h^{y_2})$

$h = \text{Hash}(\text{stock ex ...}$   
 $1.1.2022)$



## Failstop one-time signature

- $\text{Sign}(\text{SK}, m) = (\sigma_1(\text{SK}, m), \sigma_2(\text{SK}, m))$  where
- $\sigma_1(\text{SK}, m) = x_1 + m \cdot y_1 \bmod q$
- $\sigma_2(\text{SK}, m) = x_2 + m \cdot y_2 \bmod q$

## Failstop signature verification

if  $\text{PK} = (p_1, p_2)$  then the signature is valid iff

$$p_1 \cdot p_2^m = g^{\sigma_1} \cdot h^{\sigma_2}$$



## Security concept

- there are  $q$  solutions for  $\sigma_1, \sigma_2$
- an adversary breaking  $p_1, p_2$  may have valid keys, can use them, but then the legitimate user can derive  $\log_g h$

fixed for  $m$  and key  $\rightarrow$

$$P_1 \cdot P_2 = g^{\sigma_1} h^{\sigma_2}$$

$g^{x_1} h^{x_2}, g^{y_1} h^{y_2}$

$$g^{\sigma_1'} \cdot h^{\sigma_2'}$$

=

$$g^{\sigma_1} \cdot h^{\sigma_2}$$

$$h^A$$

$$= g^{-B}$$

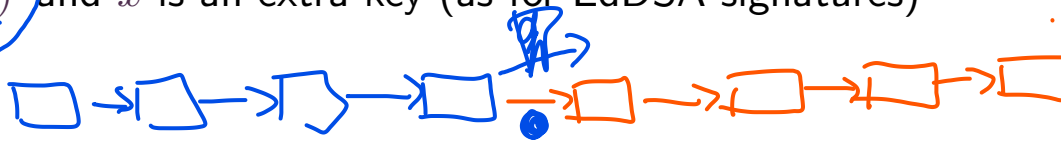
$$h = g^{B/A}$$



## Commitment to ephemeral values

- signature  $i$  contains a commitment to  $r_{\text{next}} = g^{k_{\text{next}}}$  used in the next signature. E.g., the signature is over  $M \parallel \text{Hash}(r_{\text{next}})$  instead of  $M$
- the next signature uses  $r = r_{\text{next}}$
- in order to remember  $r_{\text{next}}$  one can design a scheme where  $r_i = g^{k_i}$  where

$k_i := \text{Hash}(x, i)$  and  $x$  is an extra key (as for EdDSA signatures)



### Situation:

- the  $i$ th signature created by a clone and the  $i$ th signature created by the original device - use the same  $k_i$
- the same  $k_i$  for different messages  $\Rightarrow$  secret key gets exposed
- so: using a clone reveals the fact that the key is compromised