

## CRYPTOGRAPHY, 2004 Assignments, list # 2

1. Write a pseudocode for finding inverse of a number  $a$  modulo  $n$ . Implement such an algorithm so that no division is used – this is necessary for reducing the execution time. (Such an algorithm exists and is called *binary gcd*.)
2. Consider the tasks of computing  $m^e \bmod n$  for a fixed parameter  $e$ . Assume that the binary representation of  $e$  contains only a few ones. Which exponentiation algorithm is best suited for this case?
3. Let  $n$  be a RSA number. Is it true that every  $a < n$  has 4 square roots?
4. Let  $n$  be a RSA number. Let  $e < \phi(n)$  be an arbitrary number coprime with  $\phi(n)$ . Given  $a < n$ . How many roots of degree  $e$  of  $a$  exist? Discuss all cases.
5. Estimate the number of  $k < n$  such that  $k$  has at least one divisor less than  $B$ . Based on this estimation discuss how much influence on the runtime of primality test has preliminary trial divisions by small prime numbers.
6. Estimate the probability that Miller-Rabin primality test presented during the last lecture finds an appropriate witness for an RSA number  $n$  (i.e. a witness that proves  $n$  to be composite).

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