

1. Assume that you have a hash function $H : \{0, 1\}^{512} \rightarrow \{0, 1\}^{160}$. However, for your application, you need a function $H' : \{0, \dots, p-1\} \rightarrow \{0, \dots, q-1\}$ for some prime numbers p and q . How to customize H to get H' that has similar properties?
2. Consider the Meyer-Davies scheme. The encryption function E encrypts 80-bit blocks.
 - evaluate the complexity of finding a collision for this scheme applied directly for E (an estimation is enough),
 - adjust it (how?) to create secure hashes of length 160.
3. You need to create a key for your cryptocurrency wallet – a random 256 bit number that will be used for signing. You have a hardware cryptocurrency wallet D . The device D can import a key, but is protected against exporting the key.

You are afraid that your cryptocurrency assets will be lost once D crashes (every device eventually crashes in an unexpected moment). So you keep a paper copy of the key.

There are challenges: if you have a long sequence of bits, then it is very likely that what you input is not the same as what you have on the paper copy.

 - Design a scheme for codes on paper that enable easy and reliable recovery from a security copy on paper to the secret signing key (mnemonic codes).
Be aware of cultural threats (for example, a Chinese will never enter a digit 4 into the code, Polish guys are likely to use curse words, ...)
 - check how the problem has been solved by the BIP-39 standard used in cryptocurrency communities.
4. Consider the function $F(x, y, z) = (x \wedge y) \vee (\bar{x} \wedge z)$ used by MD5. What would be the effect of replacing it with $F(x, y, z) = (x \wedge y) \vee (x \wedge z)$ in the context of finding MD5 collisions?
5. Consider a modification of MD5 where each 32-bit block of message is used only once (it is equal to exactly one w_j). What would be the consequences for finding a collision?

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