On Alarm Protocol in Wireless Sensor Networks

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Abstract. We consider the problem of efficient alarm protocol for ad-hoc radio networks consisting of devices that try to gain access for transmission through a shared radio communication channel. The problem arise in tasks that sensors have to quickly inform the target user about an alert situation such as presence of fire, dangerous radiation, seismic vibrations, and more. In this paper, we present a protocol which uses $O(\log n)$ time slots and show that $\Omega(\log n/\log\log n)$ is a lower bound for used time slots.

Keywords: wireless sensor network, radio channel, alarm protocol, oblivious leader election

1 Introduction

We consider wireless sensor networks (WSNs) consisting of small programmable devices equipped with radio-enabled sensing capabilities and have been applied in information gathering ranging from the environment temperature, radiation, the presence of fire, seismic vibrations, and more. WSNs compared with wired networks provide many advantages in the deployment, cost and size. Wireless technology enables users to set up a network quickly, more it enables them to set up a network where it is inconvenient or impossible to wire cables. Moreover, common WSNs can consist of up to several hundreds of those small devices.

The most straightforward application of a WSN is to monitor remote or hostile environments. For example, a remote forest area can be monitored by deploying hundreds of sensors that configure themselves to form a network and immediately report upon detection of any event such as fire. Moreover, such networks can be easily extended by simply adding more devices without any rework or complex reconfiguration. The sensor nodes can ideally run for over a year on a single set of batteries. Given the cost of these sensor nodes, it is not feasible to discard dead sensor nodes, and it is also not possible to replace the batteries on these sensor nodes. Hence, there is a great need for energy-efficient protocols that can greatly reduce power consumption and increase the lifetime of wireless sensor nodes.

In this paper, we address the problem of designing protocol for an alert situation observed by the sensor nodes in a WSN and sending this information toward the sink that acts as a collector and an interface to the external world. The traffic is usually forwarded over multi-hops, that is, each node acts as a relay/router for some nodes farther away to the sink. However, in this paper we restrict our attention to a quarter size sensors e.g. MICA2DOT. We even assume that such sensors cannot listen to the channel or receive messages of any kind, also they have a very limited energy supply. Due to this restriction sensors are incapable of forwarding messages. Therefore, we have to assume that the network of such sensors is single-hop or we have two classes of wireless sensors; the first class of sensors are devices equipped with full communication features such as receiving, sending and forwarding messages, and the second class of sensors are our very weak tiny devices, capable only of sensing and sending information in a single-hop fashion. For the first class of sensors, which is the backbone of the network, we can use well known convergecast algorithms [1–5]. In this paper we deal with the second class of devices.

In Sect. 3 we shall consider the sequence $p = ((\frac{1}{2})^i)_{i=0,\ldots,L}$ of probabilities of length $L = \lceil \log_2(n) \rceil + 1$, where n denote the number of sensors and we assume that at ith slot stations try to transmit with probability $(\frac{1}{2})^i$. Let $\mathrm{SCC}_{n,k}$ denotes the event of the successful transmission when $k \in \{1, \ldots, n\}$ stations are activated and let $\Pr[\mathrm{SCC}_n] = \min\{\Pr[\mathrm{SCC}_{n,k}] : k = 1, \ldots, n\}$. The same sequence was investigated in a series of papers of Nakano and Olariu. In 2000 in [6] authors claimed that $\Pr[\mathrm{SCC}_n] \geq 0.6$ but they omit the proof (due to page limitations). In 2001 in the next paper [7] authors sketch a proof of inequality $\Pr[\mathrm{SCC}_n] \geq 1/(2\sqrt{e}) \approx 0.303$. However they do not observe that the inequality $(1-x)^{n-1} > e^{-nx}$, which is used in the proof, does not hold for all $x \in [0, 1]$ and $n > 1$. Next, in 2002 in [8] they claim that $\Pr[\mathrm{SCC}_n] \geq 0.5$ and for the proof they refer to the previous paper [6]. Let us remark that numerical calculation for small values of n (say $n < 1000$) confirms that $\Pr[\mathrm{SCC}_n] \geq 0.6$.

In Sect.3 we shall prove that $\Pr[\mathrm{SCC}_n] > 0.575$ and to the best of our knowledge this will be the first published proof of this fact. In Sect. 4 we analyze the lower bound on number of slots required by our protocols for successful transmission of alert messages with a controllable probability of success and we show that $\Omega(\log n / \log\log n)$ is asymptotically a lower bound for the number of necessary time slots. We will use the following simple lemma which we leave without proof:

Lemma 1. *Suppose A is an event, $f \geq 1$, $\Pr[A] \geq \lambda > 0$ and let $A_1, \ldots, A_m$ be independent copies of A. Then*

$$\left(m \geq \frac{\log f}{\log \frac{1}{1-\lambda}} \right) \longrightarrow \Pr[A_1 \cup \ldots \cup A_m] \geq 1 - \frac{1}{f} \ .$$

Related work. The alarm problem is similar to the wake-up problem [9–11]. In the wake-up problem it is assumed that any subset of sensors wake up spontaneously at arbitrary times and awake the remaining sensors. However, in the alarm problem it is not important to inform other sensors about a dangerous situation. Only the sink should be informed as soon as possible. Therefore, we want to design such a protocol, which is able to inform the sink with a minimal time complexity. Our considerations are directly related to the previously mentioned papers [6–8].

Algorithm 1 Randomized Alarm Algorithm RAA(n,f,T_0,Δ)

if NOT ALERT **then**
    EXIT
end if
wait until $(GetTime() \geq T_0)$
SendMessage()
$L := \lceil \log_2(n) \rceil + 1$
$r := \lceil 1.1553 \log f \rceil$
for $j := 0$ to $r - 1$ **do**
    for $i := 1$ to L **do**
        wait until $(GetTime() \geq T_0 + (i + j * L) * \Delta)$
        if $(Random() < \max(1/n, (1/2)^i))$ **then**
            SendMessage()
        end if
    end for
end for

2 Model Description

We consider a *wireless sensor network* consisting of n processing units, called *sensors* with limited power and one distinguished station called the *sink* with an unlimited power. The sensors communicate directly with the sink through a shared radio channel and a transmission succeeds if exactly one station sends at a time. We assume that sensors **can only send messages** and that they cannot listen or recognize the state of the channel. We consider only *single-hop* networks in which each station can directly communicate with the sink through a shared communication channel. We also assume that stations are synchronized and that the time is divided into short time-slots $S_0, \ldots, S_L$ of the same length Δ. There is also a fixed vector $p_0, \ldots, p_L$ of probabilities.

Let $A \subseteq \{1, \ldots, n\}$ be a set of sensors which detect an alert and let $k = |A|$. Each sensor from the set A try to send an alert message in the ith slot S_i independently with probability p_i. The transmission will be successful if in some slot S_i precisely one sensor from A will transmit. Nakano and Olariu (see [6]) call this variant of leader election algorithm an oblivious one: all stations use the same probabilities which are fixed beforehand and does not depend on the history.

Our goal is to find a reasonable small L and a vector $p_0, \ldots, p_L$ of probabilities which will guarantee a successful transmission of an alert with a probability at least $1 - \frac{1}{f}$ where $f > 1$ is a given fixed parameter and $1 \leq k \leq n$ is arbitrary.

3 Upper Bound

By $n \geq 1$ we denote the number of stations. We divide time into $L + 1$ slots. At ith time slot each station decides to transmit the alert message independently with the probability $p_i = (1/2)^i$ for $i = 0, \ldots, L$. Let $\mathrm{SCC}_{L,n,k}$ denote the event of the successful transmission when $k \in \{1,\ldots,n\}$ stations are activated. Then $\Pr[\mathrm{SCC}_{L,n,1}] = 1$ and

$$\Pr[\mathrm{SCC}_{L,n,k}] = 1 - \prod_{i=0}^{L} \left(1 - \binom{k}{1}\frac{1}{2^i}\left(1-\frac{1}{2^i}\right)^{k-1}\right)$$

for $k > 1$. Finally we put

$$\Pr[\mathrm{SCC}_{L,n}] = \min\{\Pr[\mathrm{SCC}_{L,n,k}] : k = 1,\ldots,n\} \ .$$

Theorem 1. *If* $L = \lceil \log_2 n \rceil + 1$ *then*

$$\Pr[\mathrm{SCC}_{L,n}] \geq 1 - \frac{3}{4}(1-\frac{1}{2}e^{-1/2})(1-\frac{1}{4}e^{-1/4}) \approx 0.579 \ .$$

Proof. Let $\lambda = 1 - \frac{3}{4}(1-\frac{1}{2}e^{-1/2})(1-\frac{1}{4}e^{-1/4})$. Notice that $\lambda \approx 0.579$. Let us fix $k \in \{1,\ldots,n\}$. Then there exists $i \in \{0,\ldots,L-1\}$ such that

$$2^{i-1} < k \leq 2^i \ . \tag{1}$$

We shall consider the following three cases separately: $i = 0$, $i = 1$ and $2 \leq i \leq L-1$.

Case 1: If $i = 0$ then $(2^{i-1}, 2^i] = (1/2, 1]$ so $k = 1$ and $\Pr[\mathrm{SCC}_{L,n,1}] = 1 > \lambda$ for all $n \geq 1$.

Case 2: If $i = 1$ then $(2^{i-1}, 2^i] = (1, 2]$, therefore $k = 2$ and

$$\Pr[\mathrm{SCC}_{L,n,2}] \geq 1 - (1-f_1(2))\cdot(1-f_2(2)) = 1 - \frac{5}{16} = \frac{11}{16}$$

for $n \geq 2$. Notice that $11/16 = 0.6875 > \lambda$.

Case 3: Suppose that $2 \leq i \leq L-1$. Let us consider functions

$$f_j(k) = k \cdot \frac{1}{2^j} \cdot \left(1 - \frac{1}{2^j}\right)^{k-1} \qquad (k \geq 1,\ 1 \leq j \leq L) \ .$$

If $j > 0$ then the function f_j is unimodal (with the maximum at the point $k = 1/\log(1/(1-(1/2)^j))$) hence the minimum of the function f_j on interval $(2^{i-1}, 2^i]$ is achieved at one of the edges of this interval.

Let $l_j(i) = f_j(2^{i-1})$ and $r_j(i) = f_j(2^i)$ for $j = i-1, i, i+1$. From the inequality $(1-1/x)^x \leq 1/e$ we get

$$\frac{r_{i-1}(i)}{l_{i-1}(i)} = 2\left(1 - 2^{-(i-1)}\right)^{2^{i-1}} \leq \frac{2}{e} \ .$$

On the other hand we have

$$\frac{l_i(i)}{r_i(i)} = \frac{1}{2}\left(1-2^{-i}\right)^{2^i\cdot(-1/2)}, \quad \frac{l_{i+1}(i)}{r_{i+1}(i)} = \frac{1}{4}\left(1-2^{-i}\right)^{2^i\cdot(-1/4)} \ .$$

Notice that those functions are decreasing, so the maximum is achieved for $i = 2$. Thus, $l_i(i)/r_i(i) \leq 8/9 < 1$, $l_{i+1}(i)/r_{i+1}(i) \leq 32/49 < 1$ for $i \geq 2$. Therefore, we deduce that minimum of the functions $f_{i-1}(x)$, $f_i(x)$, $f_{i+1}(x)$ on the interval are achieved at points 2^i, 2^{i-1}, 2^{i-1} respectively, and are equal to $r_{i-1}(i)$, $l_i(i)$, $l_{i+1}(i)$ i.e. $f_{i-1}(2^i)$, $f_i(2^{i-1})$, $f_{i+1}(2^{i-1})$.

Next, we notice that the functions $l_x(x), l_{x+1}(x)$ are decreasing and $r_{x-1}(x)$ is increasing for $x \geq 2$. This can be checked by inspecting the sign of the derivative (see Appendix A). Moreover

$$\lim_{x\to\infty} l_x(x) = \lim_{x\to\infty} \frac{1}{2}\left(1 - 2^{-x}\right)^{2^{x-1}-1} = \frac{1}{2}e^{-1/2} \, .$$

Hence $l_x(x) > (1/2)e^{-1/2}$ and therefore for each $u \in (2^{i-1}, 2^i]$ we have $f_i(u) > (1/2)e^{-1/2}$. Similarly, we have

$$\lim_{x\to\infty} l_{x+1}(x) = \frac{1}{4}e^{-\frac{1}{4}}, \qquad r_{x-1}(x) \geq \frac{1}{4} \text{ for } x \geq 2$$

and for each $u \in (2^{i-1}, 2^i]$ we have $f_{i-1}(u) \geq \frac{1}{4}$ and $f_{i+1}(u) > \frac{1}{4}e^{-\frac{1}{4}}$. Notice that $\Pr[\mathrm{SCC}_{L,n,k}]$ is greater than or equal to

$$1 - (1 - f_{i-1}(k))(1 - f_i(k))(1 - f_{i+1}(k))$$

for $2 \leq i \leq L - 1$ and $0 \leq f_j(k) \leq 1$. Therefore theorem is proved. □

By Thm. 1 we are able successfully send an alert message with a probability at least $1 - \frac{3}{4}\left(1 - \frac{1}{2}e^{-1/2}\right)\left(1 - \frac{1}{4}e^{-1/4}\right) \approx 0.579$ in $\lceil \log_2 n \rceil + 2$ time-slots. However, we are interested in sending an alert message with probability at least $1 - \frac{1}{f}$ for some fixed $f > 1$. We shall achieve this goal by repeating the sequence $\left((\frac{1}{2})^i\right)_{i=1,\ldots,\lceil \log_2 n \rceil + 1}$ a sufficient number of times to obtain the needed probability of success. Namely, Lemma 1 implies that a sufficient total number of time-slots required to send an alert message with probability at least $1 - \frac{1}{f}$ is equal to

$$\left\lceil \frac{\log f}{\log \frac{1}{1-\lambda}} \right\rceil \cdot (\lceil \log_2 n \rceil + 1) + 1 \approx 1.1553 \cdot \log f \cdot (\lceil \log_2 n \rceil + 1) + 1 \qquad (2)$$

where $\lambda = 1 - \frac{3}{4}\left(1 - \frac{1}{2}e^{-1/2}\right)\left(1 - \frac{1}{4}e^{-1/4}\right)$.

Based on the above discussion we build a Randomized Alarm Algorithm (see Algorithm 1). The small correction of probabilities in line 11 of its pseudo-code is motivated by Lemma 2 from the next section. The following theorem summarize its basic property:

Theorem 2. *For each $n \geq 1$ and $f > 1$ the Randomized Alarm Algorithm* RAA *sends successfully an alert message in*

$$\lceil 1.1553 \cdot \log f \rceil \cdot (\lceil \log_2 n \rceil + 1) + 1$$

time slots with probability at least $1 - \frac{1}{f}$ for arbitrary number of activated stations.

It is worth to mention that in $\mathrm{RAA}(n, f, T_0, \Delta)$ algorithm each station which want to transmit an alert message sent a signal in no more than $2\lceil 1.1553 \cdot \log f \rceil$ time-slots on average.

4 Lower bound

Let $\boldsymbol{p} = (p_i)_{i=1,\ldots,L}$ be a vector of probabilities. By $\mathrm{SCC}(\boldsymbol{p}, k)$ we denote the event of successful transmission of an alert message when k of n stations tries to transmit using the vector of probabilities $\boldsymbol{p}$.

Lemma 2. *Let $\boldsymbol{p} = (p_i)_{i=1,\ldots,L}$ be a vector of probabilities, let $q_i = \max\{p_i, \frac{1}{n}\}$ and let $\boldsymbol{q} = (q_i)_{i=1,\ldots,L}$. Then*

$$(\forall k \in \{1, \ldots, n\})(\Pr[\mathrm{SCC}(\boldsymbol{p}, k)] \leq \Pr[\mathrm{SCC}(\boldsymbol{q}, k)]) .$$

Proof. Let us fix a number $k \geq 1$ and let $f_k(p) = kp(1-p)^{k-1}$. The function f_k is unimodal, reaches a maximum at point $p = \frac{1}{k}$. Hence if $k \leq n \leq \frac{1}{p}$ then $f_k(p) \leq f_k(\frac{1}{n})$. □

We shall prove the following theorem:

Theorem 3. *If $\boldsymbol{p} = (p_i)_{i=1,\ldots,L}$ is an arbitrary vector of probabilities then there exists $k \in \{1, \ldots, n\}$ such that*

$$\Pr[\mathrm{SCC}(\boldsymbol{p}, k)] \leq 1 - \left(1 - \frac{3e}{n^{\frac{1}{2(L+1)}}}\right)^L .$$

Proof. Let us fix n and let us consider a sequence $\boldsymbol{p}$ of length L such that

$$\min_{1 \leq k \leq n} \Pr[\mathrm{SCC}(\boldsymbol{p}, k)] = \sup_{\boldsymbol{x} \in [0,1]^L} \min_{1 \leq k \leq n} \Pr[\mathrm{SCC}(\boldsymbol{x}, k)] .$$

Using Lemma 2 we may assume that $p_i \geq \frac{1}{n}$ for all $i \in \{1, \ldots, L\}$. We may also assume that $p_1 \geq p_2 \geq \ldots \geq p_L$. We additionally put $p_0 = 1$ and $p_{L+1} = 1/n$.

Lemma 3. *There exists $i \in \{0, \ldots, L\}$ such that*

$$\frac{p_i}{p_{i+1}} \geq n^{\frac{1}{L+1}} .$$

Proof. Suppose that $p_0/p_1 < n^{1/(L+1)}$, $p_1/p_2 < n^{1/(L+1)}$, $\ldots$, $p_L/p_{L+1} < n^{1/(L+1)}$. Then

$$n = \frac{p_0}{p_1} \cdot \frac{p_1}{p_2} \cdots \frac{p_L}{p_{L+1}} < n^{\frac{L+1}{L+1}} = n ,$$

what is impossible. □

Let us fix a such that $\frac{p_a}{p_{a+1}} \geq n^{\frac{1}{L+1}}$. We shall consider three cases separately: $0 < a < L$, $a = 0$ and $a = L$. In the next considerations we shall use several times the inequality $x/e^x < 1.5/x^2$ which holds for all $x > 0$ and the inequality $(1-x)^{1/x} < e^{-1}$ which holds for all $x \in (0,1)$.

Case 1: $0 < a < L$. Let $p = p_a$ and $q = p_{a+1}$. We choose $k = 1/\sqrt{pq}$. Notice that $p/q \geq n^{1/(L+1)}$, $kp = \sqrt{p/q}$ and $k^2 \geq n^{1/(L+1)}$ (because: $k^2 = (pq)^{-1} = p^{-2}(p/q) \geq p^{-2}n^{1/(L+1)} \geq n^{1/(L+1)}$). Let $k^* = \lceil k \rceil$. Then for arbitrary $x \in (0,1)$ we have $k^*x(1-x)^{k^*-1} \leq 2kx(1-x)^{k-1}$.

Subcase 1. If $i \leq a$ and $p \leq 1 - \frac{1}{e}$ then we have

$$k^*p_i(1-p_i)^{k^*-1} \leq 2kp_i(1-p_i)^{k-1} \leq 2kp(1-p)^{k-1} \leq 2kp(1-p)^k e =$$

$$\sqrt{\frac{p}{q}}(1-p)^{\frac{1}{p}\sqrt{\frac{p}{q}}}2e < \frac{\sqrt{\frac{p}{q}}}{\exp(\sqrt{\frac{p}{q}})}2e < \frac{3e}{n^{1/(L+1)}}$$

Subcase 2. If $i \leq a$ and $p > 1 - \frac{1}{e}$ then we have

$$k^*p_i(1-p_i)^{k^*-1} \leq 2kp_i(1-p_i)^{k-1} \leq 2kp(1-p)^{k-1} <$$

$$2kp\left(\frac{1}{e}\right)^{k-1} \leq 2\frac{ke}{e^k} < \frac{3e}{k^2} \leq \frac{3e}{n^{1/(L+1)}}$$

Subcase 3. If $a < i \leq L$ and $q \leq 1 - \frac{1}{e}$ then

$$k^*p_i(1-p_i)^{k^*-1} \leq 2kp_i(1-p_i)^{k-1} \leq 2kq(1-q)^{k-1} \leq 2kq(1-q)^k e =$$

$$2\sqrt{\frac{q}{p}}(1-q)^{\frac{1}{q}\sqrt{\frac{q}{p}}}e < 2\frac{\sqrt{\frac{q}{p}}}{\exp(\sqrt{\frac{q}{p}})}e < 2\sqrt{\frac{q}{p}}e \leq \frac{2e}{n^{1/(2(L+1))}}$$

Subcase 4. If $a < i \leq L$ and $q > 1 - \frac{1}{e}$ then

$$k^*p_i(1-p_i)^{k^*-1} \leq 2kp_i(1-p_i)^{k-1} \leq 2kq(1-q)^{k-1} < 2kq\left(\frac{1}{e}\right)^{k-1} \leq$$

$$\frac{2ke}{\exp(k)} < \frac{3e}{k^2} \leq \frac{3e}{n^{1/(L+1)}}$$

Therefore we shown that in all subcases of Case 1 we have

$$\Pr[\mathrm{SCC}_{k^*}] = 1 - \prod_{i=1}^{L}(1 - k^*p_i(1-p_i)^{k^*-1}) < 1 - (1 - \frac{3e}{n^{1/(2(L+1))}})^L$$

Case 2: $a = 0$. In this case we take $k = 1$ and since $p_1 \leq 1/n^{1/(L+1)}$ we get

$$\Pr[\mathrm{SCC}_1] = 1 - \prod_{i=1}^{L}(1 - 1 \cdot p_i(1-p_i)^{1-1}) \leq$$

$$1 - (1 - \frac{1}{n^{1/(L+1)}})^L < 1 - (1 - \frac{2e}{n^{1/(2(L+1))}})^L .$$

Case 3: $a = L$. In this case we take $k = n$. Then $np_L \geq n^{1/(L+1)}$. If $p_L \leq 1 - \frac{1}{e}$ we have

$$np_L(1-p_L)^{n-1} \leq np_L(1-p_L)^n e \leq np_L(1-p_L)^{\frac{1}{p_L}np_L} e < \frac{np_L}{\exp(np_L)} e < \frac{2e}{n^{1/(L+1)}}$$

and if $p_L > 1 - \frac{1}{e}$ then

$$np_L(1-p_L)^{n-1} < np_L \left(\frac{1}{e}\right)^{n-1} \leq \frac{n}{\exp(n)} e < \frac{2e}{n^{1/(L+1)}} ,$$

therefore

$$\Pr[\mathrm{SCC}_n] = 1 - \prod_{i=1}^{L}(1 - np_i(1-p_i)^{n-1}) \leq 1 - \left(1 - \frac{2e}{n^{1/(L+1)}}\right)^L < 1 - \left(1 - \frac{3e}{n^{1/(2(L+1))}}\right)^L .$$

Hence we have finished the analysis of cases and we see in each case we are able to find $k \in \{1, \ldots, n\}$ such that

$$\Pr[\mathrm{SCC}_k] < 1 - \left(1 - \frac{3e}{n^{1/(2(L+1))}}\right)^L .$$

□

Let $\mathcal{W}$ denote the main branch of the Lambert function. Let us consider an arbitrary vector $\boldsymbol{p} = (p_i)_{i=1,\ldots,L}$ of probabilities of length L. Let $\mathrm{SCC}_{L,n,k}$ denotes the event of successful transmission when k sensors are activated and let $\Pr[\mathrm{SCC}_{L,n}] = \min\{\Pr[\mathrm{SCC}_{L,n,k}] : k = 1, \ldots, n\}$.

Theorem 4. *If* $L \leq \frac{\log n}{2\log(3e)} - 1$, $f > 1$ *and* $\Pr[\mathrm{SCC}_{L,n}] > 1 - \frac{1}{f}$ *then*

$$L \geq \frac{\log n}{2\mathcal{W}\left(\frac{3e}{2}\frac{f}{f-1}\log n\right)} - 1 .$$

Proof. If $L \leq \frac{\log n}{2\log(3e)} - 1$ then $3e/n^{1/(2(L+1)} \leq 1$ so we may apply the classical Bernoulli inequality $((\forall x \leq 1)((1-x)^n \geq 1 - nx))$ to Theorem 3 and obtain the following inequality

$$\Pr[\mathrm{SCC}_{L,n}] < \frac{3eL}{n^{\frac{1}{2(L+1)}}} .$$

Hence from $\Pr[\mathrm{SCC}_{L,n}] > 1 - \frac{1}{f}$ we deduce that $3eLn^{\frac{-1}{2(L+1)}} > 1 - \frac{1}{f}$, so also $3e(L+1)n^{\frac{-1}{2(L+1)}} > 1 - \frac{1}{f}$. This inequality may be solved by the use of the Lambert function $\mathcal{W}$, giving us the required inequality. □

Let us recall that $\log x - \log\log x < \mathcal{W}(x) < \log x - \frac{1}{2}\log\log x$ for $x \geq e$ (see e.g. [12]). Using this bounds we get

$$\frac{\log n}{2\mathcal{W}\left(\frac{3e}{2}\frac{f}{f-1}\log n\right)} > \frac{\log n}{2\log\left(\frac{3e}{2}\frac{f}{f-1}\log n\right)} = \frac{1}{2}\frac{\log n}{\log\log n + \log\left(\frac{3e}{2}\frac{f}{f-1}\right)} .$$

If $f > 1$ is fixed and n tends to infinity then

$$\frac{\log n}{2\mathcal{W}\left(\frac{3}{2}e\frac{f}{f-1}\log n\right)} \sim \frac{\log n}{2\log\log n} .$$

Let us finally remark that if $f = n$ then the inequality

$$1 - \left(1 - \frac{3e}{n^{1/(2(L+1))}}\right)^{L+1} > 1 - \frac{1}{f}$$

can be solved precisely giving us a bound $L > 0.236594\log n - 1$.

5 Conclusions

In this paper we show that there exists an oblivious alarm protocol for sensor network which use $O(\log n)$ time slots and that each oblivious alarm protocol for sensor network requires $\Omega(\frac{\log n}{\log\log n})$ time slot. The algorithmic gap remains to be clarified.

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A Monotonicity of Functions from Section 3

In this appendix we prove monotonicity of functions considered in Sect. 3.

Lemma 4. *Let $f_i(k) = k\frac{1}{2^i}(1-\frac{1}{2^i})^{k-1}$. Then, the functions $f_i(2^{i-1})$, $f_{i+1}(2^{i-1})$ are decreasing and $f_{i-1}(2^i)$ is increasing for $i \geq 2$.*

Proof. Let $g_\alpha(x) = (1-\frac{1}{\alpha x})^{x-1}$. Then $f_i(2^{i-1}) = \frac{1}{2}g_2(2^{i-1})$, $f_{i+1}(2^{i-1}) = \frac{1}{4}g_4(2^{i-1})$ and $f_{i-1}(2^i) = 2g_{\frac{1}{2}}(2^i)$. Notice that

$$\frac{d}{dx}g_\alpha(x) = \left(1-\frac{1}{\alpha x}\right)^{x-1} \cdot \left(\frac{1}{x} + \frac{\alpha-1}{1-\alpha x} + \log\left(1-\frac{1}{\alpha x}\right)\right) .$$

We consider $x \geq 4$ and $\alpha \geq \frac{1}{2}$. Then $1 - 1/(\alpha x) > 0$, so $\left(1-\frac{1}{\alpha x}\right)^{x-1} > 0$. We are interested in the sign of derivative of the function g_α, so we only need to check the sign of the remaining part of the derivative. Let $z = \frac{1}{\alpha x}$. Then $0 < z < \frac{1}{2}$ and

$$\frac{1}{x} + \frac{\alpha-1}{1-\alpha x} + \log\left(1-\frac{1}{\alpha x}\right) = \alpha z + (\alpha-1)\frac{z}{z-1} - \log\left(\frac{1}{1-z}\right) .$$

We expand the right side of this equation and obtain

$$\alpha z - (\alpha-1)\sum_{i=1}^{\infty} z^i - \sum_{i=1}^{\infty}\frac{z^i}{i} = \alpha z - \sum_{i=1}^{\infty}(\alpha - 1 + \frac{1}{i})z^i = -\sum_{i=2}^{\infty}(\alpha-1+\frac{1}{i})z^i .$$

The last formula implies that if $\alpha = 1/2$ then this series is greater than zero and for $\alpha \geq 1$ this series is less than zero. □