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A compendium of NP optimization problems

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Summary. Due to the fact that no NP-complete problem can be solved in polynomial time (unless P=NP), many approximability results (both positive and negative) of NP-hard optimization problems have appeared in the technical literature. In this compendium, we collect together a large number of these results.

In the following we refer to standard complexity classes (see [Johnson, 1990]). We recall that a function $t(n)$ is 'quasi-polynomial' if a constant c exists such that $t(n) \leq n^{\log^c n}$ and we denote by QP, QNP, and QR the analogues of the usual complexity classes in the quasi-polynomial time domain.

1. NPO Problems: Definitions and Preliminaries

The basic ingredients of an optimization problem are the set of instances or input objects, the set of feasible solutions or output objects associated to any instance, and the measure defined for any feasible solution. On the analogy of the theory of NP-completeness, we are interested in studying a class of optimization problems whose feasible solutions are short and easy-to-recognize. To this aim, suitable constraints have to be introduced. We thus give the following definition.

Definition 1. *An* NP optimization problem *A is a fourtuple $(I, sol, m, goal)$ such that*

1. *I is the set of the* instances *of A and it is recognizable in polynomial time.*
2. *Given an instance x of I, $sol(x)$ denotes the set of* feasible solutions *of x. These solutions are short, that is, a polynomial p exists such that, for any $y \in sol(x)$, $|y| \leq p(|x|)$. Moreover, it is decidable in polynomial time whether, for any x and for any y such that $|y| \leq p(|x|)$, $y \in sol(x)$.*
3. *Given an instance x and a feasible solution y of x, $m(x, y)$ denotes the positive integer* measure *of y. The function m is computable in polynomial time and is also called the* objective *function.*
4. *$goal \in \{\max, \min\}$.*

The *class* NPO is the set of all NP optimization problems.

The goal of an NPO problem with respect to an instance x is to find an *optimum solution*, that is, a feasible solution y such that

$$m(x,y) = goal\{m(x,y') \ : \ y' \in sol(x)\}.$$

In the following sol^* will denote the multi-valued function mapping an instance x to the set of optimum solutions, while opt will denote the function mapping an instance x to the measure of an optimum solution.

An NPO problem is said to be *polynomially bounded* if a polynomial q exists such that, for any instance x and for any solution y of x, $m(x,y) \le q(|x|)$. The *class* NPO PB is the set of polynomially bounded NPO problems.

2. Approximate Algorithms and Approximation Classes

It is well-known that if an NPO problem can be solved in polynomial time, then its corresponding decision problem can also be solved in polynomial time. As a consequence, if P $\neq$ NP, then any NPO problem whose corresponding decision problem is NP-complete is not solvable in polynomial time. In these cases we sacrifice optimality and start looking for approximate solutions computable in polynomial time.

Definition 2. *Let A be an* NPO *problem. Given an instance x and a feasible solution y of x, we define the* performance ratio of y with respect to x *as*

$$R(x,y) = \max\left\{\frac{m(x,y)}{opt(x)}, \frac{opt(x)}{m(x,y)}\right\}.$$

The performance ratio is always a number greater than or equal to 1 and is as close to 1 as y is close to the optimum solution.

Definition 3. *Let A be an* NPO *problem and let T be an algorithm that, for any instance x of A, returns a feasible solution $T(x)$ of x. Given an arbitrary function $r : N \rightarrow (1,\infty)$, we say that T is an* $r(n)$-approximate algorithm for A *if, for any instance x, the performance ratio of the feasible solution $T(x)$ with respect to x verifies the following inequality:*

$$R(x,T(x)) \le r(|x|).$$

If an NPO problem admits an $r(n)$-approximate polynomial-time algorithm we say that it is approximable within $r(n)$.

Definition 4. *An* NPO *problem A belongs to the* class APX *if it is approximable within ε, for some constant $\varepsilon > 1$.*

Definition 5. *Let A be an* NPO *problem. An algorithm T is said to be an* approximation scheme *for A if, for any instance x of A and for any rational $\varepsilon > 1$, $T(x,\varepsilon)$ returns a feasible solution of x whose performance ratio is at most ε.*

Definition 6. *An* NPO *problem A belongs to the* class PTAS *if it admits a polynomial-time approximation scheme, that is, an approximation scheme whose time complexity is bounded by $q(|x|)$ where q is a polynomial.*

Observe that the time complexity of an approximation scheme in the above definition may be of the type $2^{1/(\varepsilon-1)}p(|x|)$ or $|x|^{1/(\varepsilon-1)}$ where p is a polynomial. Thus, computations with ε values very close to 1 may turn out to be practically unfeasible. This leads us to the following definition.

Definition 7. *An* NPO *problem A belongs to the* class FPTAS *if it admits a* fully *polynomial-time approximation scheme, that is, an approximation scheme whose time complexity is bounded by $q(|x|, 1/(\varepsilon-1))$ where q is a polynomial.*

Clearly, the following inclusions hold:

$$\text{FPTAS} \subseteq \text{PTAS} \subseteq \text{Apx} \subseteq \text{NPO}.$$

It is also easy to see that these inclusions are strict if and only if $\text{P} \neq \text{NP}$.

3. Completeness in Approximation Classes

In this section we define a natural approximation preserving reducibility and introduce the notion of completeness both in NPO and in Apx.

Definition 8. *Let A and B be two* NPO *problems. A is said to be* PTAS-reducible *to B, in symbols $A \leq_{\text{PTAS}} B$, if three functions f, g, and c exist such that:*

1. *For any $x \in I_A$ and for any rational $\varepsilon \in (1, \infty)$, $f(x, \varepsilon) \in I_B$ is computable in time polynomial with respect to $|x|$.*
2. *For any $x \in I_A$, for any $y \in sol_B(f(x, \varepsilon))$, and for any rational $\varepsilon \in (1, \infty)$, $g(x, y, \varepsilon) \in sol_A(x)$ is computable in time polynomial with respect to both $|x|$ and $|y|$.*
3. *$c : (1, \infty) \to (1, \infty)$ is computable and invertible.*
4. *For any $x \in I_A$, for any $y \in sol_B(f(x, \varepsilon))$, and for any rational $\varepsilon \in (1, \infty)$,*

$$R_B(f(x, \varepsilon), y) \leq c(\varepsilon) \text{ implies } R_A(x, g(x, y, \varepsilon)) \leq \varepsilon.$$

Remark 1. In [Papadimitriou and Yannakakis, 1991] a different kind of reducibility between optimization problems is defined which is a restriction of the PTAS-reducibility and is called L-reducibility.

It is easy to see that the previous definition satisfies the following fact.

Proposition 1. *If $A \leq_{\text{PTAS}} B$ and $B \in$ Apx (respectively, $B \in$ PTAS), then $A \in$ Apx (respectively, $A \in$ PTAS).*

Definition 9. *A problem $A \in$ NPO is* NPO-complete *if, for any $B \in$ NPO, $B \leq_{\text{PTAS}} A$.*

Analogously, we can define the notion of completeness in the class NPO PB.

Definition 10. *A problem $A \in$ NPO is* Apx-hard *if, for any $B \in$ Apx, $B \leq_{\text{PTAS}} A$. An* Apx*-hard problem is* Apx-complete *if it belongs to* Apx*.*

4. A list of NPO problems

The list contains almost 150 entries. A typical entry consists of eight parts: the first 4 parts are mandatory while the last 4 parts are optional.

1. The problem name that also specifies the goal of the problem.
2. The definition of the instances of the problem.
3. The definition of the feasible solutions of the problem.
4. The definition of the measure of a feasible solution.
5. A 'good news' part that contains the best approximation result for the problem.
6. A 'bad news' part that contains the worst approximation negative result for the problem.
7. A section of additional comments.
8. A reference to the 'closest' problem appearing in the list published in [Garey and Johnson, 1979].

The list is organized according to subject matter as done in [Garey and Johnson, 1979]. In particular the entries are divided into the following twelve categories:

GT Graph theory: 40 entries.
ND Network design: 41 entries.
SP Sets and partitions: 10 entries.
SR Storage and retrieval: 5 entries.
SS Sequencing and scheduling: 12 entries.
MP Mathematical programming: 15 entries.
AN Algebra and number theory: 1 entry.
GP Games and puzzles: no entry.
LO Logic: 13 entries.
AL Automata and language theory: 5 entries.
PO Program optimization: 1 entry.
MS Miscellaneous: 9 entry.

We have ignored problems with too obscure definitions and problems for which the membership in NP was not guaranteed. Certainly, we missed many other results. Indeed, this is the first compilation of the list and we ask everybody to help us in correcting, improving, and enlarging it!

Graph Theory

Covering and Partitioning

GT1. MINIMUM VERTEX COVER

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A vertex cover for G, i.e., a subset $V' \subseteq V$ such that, for each edge $(u,v) \in E$, at least one of u and v belongs to V'.
MEASURE: Cardinality of the vertex cover, i.e., $|V'|$.

Good News: Approximable within $2 - \frac{\log\log|V|}{2\log|V|}$ [Bar-Yehuda and Even, 1985].
Bad News: APX-complete [Papadimitriou and Yannakakis, 1991].
Comment: Transformation from bounded MAXIMUM 3-SATISFIABILITY. Admits a PTAS for planar graphs [Baker, 1994]. Variation in which each vertex has a nonnegative weight and the objective is to maximize the total weight of the vertex cover is approximable within $2 - \log\log|V|/2\log|V|$ on general graphs and within 3/2 for planar graphs [Bar-Yehuda and Even, 1985].
Variation in which the degree of G is bounded by a constant B is APX-complete [Papadimitriou and Yannakakis, 1991]. For $B = 3$ it is approximable within 5/4, for $B \geq 10$ it is approximable within $2B^2/(B^2 + B - 1)$ [Monien and Speckenmeyer, 1983]. The generalization to k-hypergraphs, for $k \geq 2$, is approximable within k [Kolaitis and Thakur, 1993]. If the vertex cover is required to be the nodes of a tree in the graph and the objective is to minimize the number of edges in the tree, the problem is approximable within 2 [Arkin, Halldórsson, and Hassin, 1993]. If the graph is edge-weighted, the vertex cover is required to be a cycle, and the objective is to minimize the weight of the edges in the cycle, the problem is approximable within 3.5 [Arkin, Halldórsson, and Hassin, 1993]. The constrained variation in which the input is extended with a positive integer k and a subset S of V, and the problem is to find the vertex cover of size k that contains the largest number of vertices from S, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: GT1

GT2. MINIMUM DOMINATING SET

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A dominating set for G, i.e., a subset $V' \subseteq V$ such that for all $u \in V - V'$ there is a $v \in V'$ for which $(u,v) \in E$.
MEASURE: Cardinality of the dominating set, i.e., $|V'|$.

Good News: Approximable within $O(\log|V|)$ by reduction to MINIMUM SET COVER [Kann, 1992b].
Bad News: Not approximable within $c\log|V|$ for any $c < 1/4$ unless NP $\subset$ QP [Lund and Yannakakis, 1993a]. Not approximable within $c\log|V|$ for any $c < 1/8$, unless NP $\subset$ DTIME$\left(|V|^{\log\log|V|}\right)$ [Bellare, Goldwasser, Lund, and Russell, 1993].
Comment: Equivalent to MINIMUM SET COVER under L-reduction. If it is NP-hard to approximate within $\omega(\log n)$, then it is complete for the class of log-approximable problems [Khanna, Motwani, Sudan, and Vazirani, 1994]. Admits a PTAS for planar graphs [Baker,

1994]. Variation in which the degree of G is bounded by a constant B is APX-complete [Papadimitriou and Yannakakis, 1991] and is approximable within $\sum_{i=1}^{B+1} \frac{1}{i}$ by reduction to MINIMUM SET COVER.
Garey and Johnson: GT2

GT3. MINIMUM EDGE DOMINATING SET

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: An edge dominating set for G, i.e., a subset $E' \subseteq E$ such that for all $e_1 \in E - E'$ there is an $e_2 \in E'$ such that e_1 and e_2 are adjacent.
MEASURE: Cardinality of the edge dominating set, i.e., $|E'|$.

Good News: Admits a PTAS for planar graphs [Baker, 1994].
Garey and Johnson: GT2

GT4. MINIMUM INDEPENDENT DOMINATING SET

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: An independent dominating set for G, i.e., a subset $V' \subseteq V$ such that for all $u \in V - V'$ there is a $v \in V'$ for which $(u, v) \in E$, and such that no two vertices in V' are joined by an edge in E.
MEASURE: Cardinality of the independent dominating set, i.e., $|V'|$.

Bad News: NPO PB-complete [Kann, 1993]. Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Halldórsson, 1993b].
Comment: The problem is also called *Minimum Maximal Indepencence Number*. Transformation from SHORTEST PATH WITH FORBIDDEN PAIRS. Variation in which the degree of G is bounded by a constant B is APX-complete [Kann, 1992b].
Garey and Johnson: GT2

GT5. MINIMUM GRAPH COLORING

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A coloring of G, i.e., a partition of V into disjoint sets $V_1, V_2, \ldots, V_k$ such that each V_i is an independent set for G.
MEASURE: Cardinality of the coloring, i.e., the number of disjoint independent sets V_i.

Good News: Approximable within $O\left(|V| \frac{(\log \log |V|)^2}{(\log |V|)^3}\right)$ [Halldórsson, 1993a].
Bad News: Not approximable within $|V|^{1/14-\varepsilon}$ for any $\varepsilon > 0$ [Bellare and Sudan, 1994].
Comment: The problem is also called *Minimum Cromatic Number*. Not approximable within $|V|^{1/10-\varepsilon}$ for any $\varepsilon > 0$, unless QNP$\subseteq$co-QR [Bellare and Sudan, 1994].
If the graph is 3-colorable the problem is approximable within $O(|V|^{0.4})$ [Blum, 1989], but it is not approximable within 5/3 [Khanna, Linial and Safra, 1993]. MINIMUM FRACTIONAL CHROMATIC NUMBER, the linear programming relaxation in which the independent sets $V_1, V_2, \ldots, V_k$ do not need to be disjoint, and in the solution every independent set V_i is assigned a nonnegative value λ_i such that for each vertex $v \in V$ the sum of the values assigned to the independent sets containing v is at most 1, and the measure is the sum

$\sum \lambda_i$, is not approximable within $|V|^c$ for some constant c [Lund and Yannakakis, 1993a]. The corresponding maximization problem, where the number of "not needed colors", i.e. $|V| - k$, is to be maximized, is approximable within 2 [Demange, Grisoni, and Paschos, 1994]. The constrained variation in which the input is extended with a positive integer k, a vertex $v_0 \in V$ and a subset S of V, and the problem is to find the k-coloring that colors the largest number of vertices from S in the same way as v_0, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: GT4

GT6. Minimum Edge Coloring

Instance: Graph $G = \langle V, E \rangle$.
Solution: A coloring of E, i.e., a partition of E into disjoint sets $E_1, E_2, \ldots, E_k$ such that, for $1 \le i \le k$, no two edges in E_i share a common endpoint in G.
Measure: Cardinality of the coloring, i.e., the number of disjoint sets E_i.

Good News: Approximable within 4/3, and even approximable with an absolute error guarantee of 1 [Nishizeki and Chiba, 1988].
Bad News: Not approximable within $4/3 - \varepsilon$ for any $\varepsilon > 0$ [Nishizeki and Chiba, 1988].
Comment: The problem is also called *Minimum Cromatic Index.*
Garey and Johnson: OPEN5

GT7. Minimum Feedback Vertex Set

Instance: Directed graph $G = \langle V, A \rangle$.
Solution: A feedback vertex set, i.e., a subset $V' \subseteq V$ such that V' contains at least one vertex from every directed cycle in G.
Measure: Cardinality of the feedback vertex set, i.e., $|V'|$.

Good News: Approximable within $4 - 2/n$ [Bar-Yehuda, Geiger, Naor, and Roth, 1994].
Bad News: Apx-complete [Kann, 1992b].
Comment: Transformations from Minimum Vertex Cover and Minimum Feedback Arc Set [Ausiello, D'Atri, and Protasi, 1980]. The variation in which a weight is assigned to each vertex it is approximable within $\min\{2\Delta^2, 4\log n\}$ where Δ denotes the maximum degree in G. This variation is approximable within 10 for planar graphs and within $4 - 2/n$ for graphs in which a prescribed subset of the vertices is not allowed to participate in any feedback vertex set [Bar-Yehuda, Geiger, Naor, and Roth, 1994]. The constrained variation in which the input is extended with a positive integer k and a subset S of V, and the problem is to find the feedback vertex set of size k that contains the largest number of vertices from S, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: GT7

GT8. Minimum Feedback Arc Set

Instance: Directed graph $G = \langle V, A \rangle$.
Solution: A feedback arc set, i.e., a subset $A' \subseteq A$ such that A' contains at least one arc from every directed cycle in G.

Measure: Cardinality of the feedback arc set, i.e., $|A'|$.

Bad News: Apx-hard [Kann, 1992b].

Comment: Transformation from Minimum Vertex Cover [Ausiello, D'Atri, and Protasi, 1980]. The constrained variation in which the input is extended with a positive integer k and a subset S of A, and the problem is to find the feedback edge set of size k that contains the largest number of edges from S, is not approximable within $|E|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993]. The complementary problem of finding the maximum set of edges A' such that $G' = \langle V, A' \rangle$ is acyclic is approximable within $2/(1 + \Omega(1/\sqrt{\Delta}))$ where Δ is the maximum degree [Berger and Shor, 1990] and it is Apx-complete [Papadimitriou and Yannakakis, 1991].

Garey and Johnson: GT8

GT9. Maximum Triangle Packing

Instance: Graph $G = \langle V, E \rangle$.

Solution: A triangle packing for G, i.e., a collection $V_1, V_2, \ldots, V_k$ of disjoint subsets of V, each containing exactly 3 vertices, such that for each $V_i = \{u_i, v_i, w_i\}$, $1 \le i \le k$, all three of the edges $\langle u_i, v_i \rangle$, $\langle u_i, w_i \rangle$, and $\langle v_i, w_i \rangle$ belong to E.

Measure: Cardinality of the triangle packing, i.e., the number of disjoint subsets V_i.

Good News: Approximable within 2 [Halldórsson, 1994].

Bad News: Apx-complete [Kann, 1991].

Comment: Transformation from bounded Maximum 3-Dimensional Matching. Admits a PTAS for planar graphs [Baker, 1994]. Variation in which the degree of G is bounded by a constant B is Apx-complete

Garey and Johnson: GT11

GT10. Maximum H-Matching

Instance: Graph $G = \langle V_G, E_G \rangle$ and a fixed graph $H = \langle V_H, E_H \rangle$ with at least three vertices in some connected component.

Solution: A H-matching for G, i.e., a collection $G_1, G_2, \ldots, G_k$ of disjoint subgraphs of G, each isomorphic to H.

Measure: Cardinality of the H-matching, i.e., the number of disjoint subgraphs G_i.

Good News: Approximable within $(|V_H| + 1)/2$ [Halldórsson, 1994].

Bad News: Apx-hard [Kann, 1994].

Comment: Transformation from bounded Maximum 3-Satisfiability. Variation in which the degree of G is bounded by a constant B is Apx-complete [Kann, 1994]. Admits a PTAS for planar graphs [Baker, 1994], but does not admit an FPTAS unless P = NP [Berman, Johnson, Leighton, Shor and Snyder, 1990]. Induced Maximum H-Matching, i.e., where the subgraphs G_i are induced subgraphs of G, has the same good and bad news as the ordinary problem, even when the degree of G is bounded.

Garey and Johnson: GT12

GT11. Minimum Clique Partition

Instance: Graph $G = \langle V, E \rangle$.

SOLUTION: A clique partition for G, i.e., a partition of V into disjoint subsets $V_1, V_2, \ldots, V_k$ such that, for $1 \leq i \leq k$, the subgraph induced by V_i is a complete graph.
MEASURE: Cardinality of the clique partition, i.e., the number of disjoint subsets V_i.

Good News: Approximable within $O\left(|V|\frac{(\log\log|V|)^2}{(\log|V|)^3}\right)$ [Halldórsson, 1993a].
Bad News: Not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Lund and Yannakakis, 1993a].
Comment: Equivalent to MINIMUM GRAPH COLORING [Paz and Moran, 1981]. The corresponding maximization problem, where $|V| - k$, is to be maximized, is approximable within 2 [Demange, Grisoni, and Paschos, 1994].
Garey and Johnson: GT15

GT12. MINIMUM k-CAPACITATED TREE PARTITION

INSTANCE: Graph $G = \langle V, E\rangle$, and a weight function $w : E \to N$.
SOLUTION: A k-capacitated tree partition of G, i.e., a collection of vertex-disjoint subsets $E_1, \ldots, E_m$ of E such that, for each i, the subgraph induced by E_i is a tree of at least k vertices.
MEASURE: The weight of the partition, i.e., $\sum_{e \in \bigcup_i E_i} w(e)$.
Good News: Approximable within $2 - 1/|V|$ [Goemans and Williamson, 1992].
Comment: The variation in which the trees must contain exactly k vertices and the triangle inequality is satisfied is approximable within $4(1 - 1/k)(1 - 1/|V|)$. Similar results hold for the corresponding cycle and path partitioning problems with the triangle inequality [Goemans and Williamson, 1992].

GT13. MINIMUM CLIQUE COVER

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: A clique cover for G, i.e., a collection $V_1, V_2, \ldots, V_k$ of subsets of V, such that each V_i induces a complete subgraph of G and such that for each edge $\langle u, v\rangle \in E$ there is some V_i that contains both u and v.
MEASURE: Cardinality of the clique cover, i.e., the number of subsets V_i.

Good News: Approximable within $O(\log|V|) \cdot f(|V|)$ if MAXIMUM CLIQUE is approximable within $f(|V|)$ [Simon, 1990].
Bad News: Not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$, unless P = NP [Lund and Yannakakis, 1993a].
Comment: Equivalent to MINIMUM CLIQUE PARTITION under ratio-preserving reduction [Kou, Stockmeyer, and Wong, 1978] and [Simon, 1990]. The corresponding maximization problem, where $|E| - k$ is to be maximized, is approximable within 2 [Demange, Grisoni, and Paschos, 1994]. The constrained variation in which the input is extended with a positive integer k, a vertex $v_0 \in V$ and a subset S of V, and the problem is to find the clique cover of size k that contains the largest number of vertices from S, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: GT17

GT14. MINIMUM COMPLETE BIPARTITE SUBGRAPH COVER

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A complete bipartite subgraph cover for G, i.e., a collection $V_1, V_2, \ldots, V_k$ of subsets of V, such that each V_i induces a complete bipartite subgraph of G and such that for each edge $\langle u, v \rangle \in E$ there is some V_i that contains both u and v.
MEASURE: Cardinality of the complete bipartite subgraph cover, i.e., the number of subsets V_i.

Good News: Approximable within $O(\log |V|) \cdot f(|V|)$ if MAXIMUM CLIQUE is approximable within $f(|V|)$ [Simon, 1990].
Bad News: Not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$, unless P = NP [Lund and Yannakakis, 1993a].
Comment: Equivalent to MINIMUM CLIQUE PARTITION under ratio-preserving reduction [Simon, 1990].
Garey and Johnson: GT18

GT15. MINIMUM VERTEX DISJOINT CYCLE COVER

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A family F of vertex disjoint cycles covering V.
MEASURE: Number of cycles in F.

Bad News: Not in APX [Sahni and Gonzalez, 1976].
Comment: Variation in which the graph G is directed is not in APX.

GT16. MINIMUM EDGE DISJOINT CYCLE COVER

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A family F of edge disjoint cycles covering V.
MEASURE: Number of cycles in F.

Bad News: Not in APX [Sahni and Gonzalez, 1976].
Comment: Variation in which the graph G is directed is not in APX.

GT17. MINIMUM CUT COVER

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A collection $V_1, \ldots, V_m$ of cuts, i.e., a collection of subsets $V_i \subseteq V$ such that, for each edge $(u, v) \in E$, a subset V_i exists such that either $u \in V_i$ and $v \notin V_i$ or $u \notin V_i$ and $v \in V_i$.
MEASURE: Cardinality of the collection, i.e., m.

Good News: Approximable within $1 + (\log |V| - 3 \log\log |V|)/opt(G)$ [Motwani and Naor, 1993].
Bad News: There is no polynomial-time algorithm with relative error less than 1.5. [Motwani and Naor, 1993].
Comment: The negative result is obtained by relating the problem with the coloring problem. Solvable in polynomial time for planar graphs. Observe that any graph has a cut cover of cardinality $\lceil \log |V| \rceil$.

Subgraphs and Supergraphs

GT18. MAXIMUM CLIQUE

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: A clique in G, i.e. a subset $V' \subseteq V$ such that every two vertices in V' are joined by an edge in E.
MEASURE: Cardinality of the clique, i.e., $|V'|$.

Good News: Approximable within $O\left(|V|/(\log|V|)^2\right)$ [Boppana and Halldórsson, 1992].
Bad News: Not approximable within $|V|^{1/6-\varepsilon}$ for any $\varepsilon > 0$ [Bellare and Sudan, 1994].
Comment: The same problem as MAXIMUM INDEPENDENT SET on the complementary graph. Not approximable within $|V|^{1/4-\varepsilon}$ for any $\varepsilon > 0$, unless QNP$\subseteq$co-QR [Bellare and Sudan, 1994].
Garey and Johnson: GT19

GT19. MAXIMUM INDEPENDENT SET

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: An independent set of vertices, i.e. a subset $V' \subseteq V$ such that no two vertices in V' are joined by an edge in E.
MEASURE: Cardinality of the independent set, i.e., $|V'|$.

Good News: See MAXIMUM CLIQUE.
Bad News: See MAXIMUM CLIQUE.
Comment: The same problem as MAXIMUM CLIQUE on the complementary graph. Admits a PTAS for planar graphs [Baker, 1994]. Variation in which the degree of G is bounded by a constant B is APX-complete [Papadimitriou and Yannakakis, 1991] and is approximable within $(B+3)/5-\varepsilon$ if B is even and within $(B+3.25)/5-\varepsilon$ if B is odd where ε is any fixed number greater than 0 [Berman and Fürer, 1994]. For large values of B it is approximable within $O(B/\log\log B)$ [Halldórsson and Radhakrishnan, 1994].
Garey and Johnson: GT20

GT20. MAXIMUM INDEPENDENT SEQUENCE

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: An independent sequence for G, i.e., a sequence $v_1, \ldots, v_m$ of independent vertices of G such that, for all $i < m$, a vertex $\bar{v}_i \in V$ exists which is connected to v_{i+1} but is not connected to any v_j for $j \leq i$.
MEASURE: Length of the sequence, i.e., m.

Bad News: Not approximable within $|V|^{\varepsilon}$ for some $\varepsilon > 0$. [Blundo, De Santis, and Vaccaro, 1994].
Comment: Transformation from MAXIMUM CLIQUE.

GT21. MAXIMUM INDUCED SUBGRAPH WITH PROPERTY Π

INSTANCE: Graph $G = \langle V, E\rangle$. The property Π must be hereditary, i.e., every subgraph of G' satisfies Π whenever G' satisfies Π, and non-trivial, i.e., it is satisfied for infinitely many graphs and false for infinitely many graphs.

SOLUTION: A subset $V' \subseteq V$ such that the subgraph induced by V' has the property Π.
MEASURE: Cardinality of the induced subgraph, i.e., $|V'|$.

Bad News: Not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ unless P=NP, if Π is false for some clique or independent set (for example planar, outerplanar, bipartite, complete bipartite, acyclic, degree-constrained, chordal, interval). Not approximable within $2^{\log^{0.5-\varepsilon}|V|}$ for any $\varepsilon > 0$ unless NP$\subset$QP, if Π is a non-trivial hereditary graph property (for example comparability, permutation, perfect, circular-arc, circle, line graph) [Lund and Yannakakis, 1993b].
Comment: The same problem on directed graphs is not approximable within $2^{\log^{0.5-\varepsilon}|V|}$ for any $\varepsilon > 0$ unless NP$\subset$QP, if Π is a non-trivial hereditary digraph property (for example acyclic, transitive, symmetric, antisymmetric, tournament, degree-constrained, line digraph) [Lund and Yannakakis, 1993b]. Admits a PTAS for planar graphs if Π is hereditary and determined by the connected components, i.e., G' satisfies Π whenever every connected component of G' satisfies Π [Nishizeki and Chiba, 1988].
Garey and Johnson: GT21

GT22. MINIMUM VERTEX DELETION TO OBTAIN SUBGRAPH WITH PROPERTY Π

INSTANCE: Directed or undirected graph $G = \langle V, E\rangle$.
SOLUTION: A subset $V' \subseteq V$ such that the subgraph induced by $V - V'$ has the property Π.
MEASURE: Cardinality of the set of deleted vertices, i.e., $|V'|$.

Good News: Approximable within some constant for any hereditary property Π with a finite number of minimal forbidden subgraphs (for example transitive digraph, symmetric, antisymmetric, tournament, line graph, and interval) [Lund and Yannakakis, 1993b]. Approximable within some constant for any property Π that can be expressed as a universal first order sentence over subsets of edges of the graph [Kolaitis and Thakur, 1991].
Bad News: APX-hard for any non-trivial hereditary property Π [Lund and Yannakakis, 1993b].
Comment: It is approximable within $O(\log|V|)$ if the subgraph has to be bipartite [Garg, Vazirani, and Yannakakis, 1994].

GT23. MINIMUM EDGE DELETION TO OBTAIN SUBGRAPH WITH PROPERTY Π

INSTANCE: Directed or undirected graph $G = \langle V, E\rangle$.
SOLUTION: A subset $E' \subseteq E$ such that the subgraph $G = \langle V, E - E'\rangle$ has the property Π.
MEASURE: Cardinality of the set of deleted edges, i.e., $|E'|$.

Good News: Approximable within some constant for any property Π that can be expressed as a universal first order sentence over subsets of edges of the graph [Kolaitis and Thakur, 1991].

GT24. MAXIMUM INDUCED CONNECTED SUBGRAPH WITH PROPERTY Π

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: A subset $V' \subseteq V$ such that the subgraph induced by V' is connected and has the property Π.
MEASURE: Cardinality of the induced connected subgraph, i.e., $|V'|$.

Bad News: Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ if Π is a non-trivial hereditary graph property that is satisfied by all paths and is false for some complete bipartite graph (for example path, tree, planar, outerplanar, bipartite, chordal, interval) [Lund and Yannakakis, 1993b].
Comment: NPO PB-complete when Π is either path or chordal [Berman and Schnitger, 1992].
Garey and Johnson: GT22 and GT23

GT25. Minimum Vertex Deletion to Obtain Connected Subgraph with Property Π

Instance: Directed or undirected graph $G = \langle V, E\rangle$.
Solution: A subset $V' \subseteq V$ such that the subgraph induced by $V - V'$ is connected and has the property Π.
Measure: Cardinality of the set of deleted vertices, i.e., $|V'|$.

Bad News: Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ if Π is any non-trivial hereditary property determined by the blocks (for example planar, outerplanar, bipartite, chordal, cactus, acyclic graph, acyclic digraph, without cycles of specified length, symmetric digraph, antisymmetric digraph) [Yannakakis, 1979].

GT26. Minimum Edge-Deletion Bipartization

Instance: Graph $G = \langle V, E\rangle$ and a weight function $w : E \to N$.
Solution: An edge-deletion bipartization, i.e., a subset $E' \subseteq E$ such that $G = \langle V, E - E'\rangle$ is bipartite.
Measure: The weight of the bipartization, i.e., $\sum_{e\in E'} w(E)$.
Good News: Approximable within $\log n$ [Garg, Vazirani, and Yannakakis, 1993b].
Bad News: Apx-hard [Garg, Vazirani, and Yannakakis, 1993b].

GT27. Maximum k-Colorable Subgraph

Instance: Graph $G = \langle V, E\rangle$.
Solution: A subset $E' \subseteq E$ such that the subgraph $G' = \langle V, E'\rangle$ is k-colorable, i.e., there is a coloring for G' of cardinality at most k.
Measure: Cardinality of the subgraph, i.e., $|E'|$.

Good News: Approximable within $\frac{k}{k-1}$ [Vitanyi, 1981].
Bad News: Apx-complete for $k \geq 2$ [Papadimitriou and Yannakakis, 1991].
Comment: Equivalent to Maximum Cut for $k = 2$.

GT28. Maximum Edge Subgraph

Instance: Graph $G = \langle V, E\rangle$ and positive integer k.
Solution: A subset $V' \subseteq V$ such that $|V'| = k$.
Measure: Cardinality of the edges in the subgraph induced by V'.

Good News: Approximable within $O(|V|^{0.3885})$ [Kortsarz and Peleg, 1993].

GT29. Minimum Edge 2-Spanner

Instance: Connected graph $G = \langle V, E\rangle$.
Solution: A 2-spanner of G, i.e., a spanning subgraph G' of G such that, for any pair of vertices u and v, the shortest path between u and v in G' is at most twice the shortest path between u and v in G.
Measure: The number of edges in G'.
Good News: Approximable within $O(\log n)$ [Kortsarz and Peleg, 1992].
Comment: The variation in which the goal is to minimize the maximum degree in G' is approximable within $O(\sqrt{\log n}\Delta^{1/4})$ where Δ is the maximum degree in G [Kortsarz and Peleg, 1994].

GT30. Maximum k-Colorable Induced Subgraph

Instance: Graph $G = \langle V, E\rangle$.
Solution: A subset $V' \subseteq V$ such that the induced subgraph $G' = \langle V', E'\rangle$ is k-colorable, i.e., there is a coloring for G' of cardinality at most k.
Measure: Cardinality of the vertex set of the induced subgraph, i.e., $|V'|$.
Bad News: As hard to approximate as Maximum Independent Set for $k \geq 1$ [Panconesi and Ranjan, 1993].
Comment: Transformation from Maximum Independent Set. Equivalent to Maximum Independent Set for $k = 1$. Admits a PTAS if G is restricted to be planar [Nishizeki and Chiba, 1988].

GT31. Minimum Equivalent Digraph

Instance: Directed graph $G = \langle V, E\rangle$.
Solution: A subset $E' \subseteq E$ such that, for every ordered pair of vertices $u, v \in V$, the graph $G' = \langle V, E'\rangle$ contains a directed path from u to v if and only if G does.
Measure: Cardinality of E', i.e., $|E'|$.

Good News: Approximable within about 1.61 [Khuller, Raghavachari, and Young, 1994b].
Garey and Johnson: GT33

GT32. Minimum Interval Graph Completion

Instance: Graph $G = \langle V, E\rangle$.
Solution: An interval graph $G' = \langle V, E'\rangle$ that contains G as a subgraph, i.e., $E \subseteq E'$. An interval graph is a graph whose vertices can be mapped to distinct intervals in the real line such that two vertices in the graph have an edge between them if and only if their corresponding intervals overlap.
Measure: The cardinality of the interval graph, i.e., $|E'|$

Good News: Approximable within $O\left(\log^2 |V|\right)$ [Ravi, Agrawal, and Klein, 1991].
Garey and Johnson: GT35

GT33. Minimum Chordal Graph Completion

Instance: Graph $G = \langle V, E \rangle$.
Solution: A chordal graph completion, i.e., a superset E' containing E such that $G' = \langle V, E' \rangle$ is chordal, that is, for every simple cycle of more than 3 vertices in G', there is some edge in E' that is not involved in the cycle but that joins two vertices in the cycle.
Measure: The size of the completion, i.e., $|E' - E|$.
Good News: Approximable within $O(|E|^{1/4} \log^{3.5} |V|)$ [Klein, Agrawal, Ravi, and Rao, 1990].
Comment: Approximable within $O(\log^4 |V|)$ for graphs with bounded degree [Klein, Agrawal, Ravi, and Rao, 1990].
Garey and Johnson: OPEN4

GT34. Maximum Constrained Hamiltonian Circuit

Instance: Graph $G = \langle V, E \rangle$ and subset $S \subseteq E$ of the edges.
Solution: A Hamiltonian circuit C in G, i.e., a circuit that visits every vertex in V once.
Measure: Cardinality of the edges in S that are used in the circuit C, i.e., $|S \cap C|$.
Bad News: Not approximable within $|E|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Comment: Variation in which the graph is directed has the same bad news [Zuckerman, 1993].
Garey and Johnson: Similar to GT37 and GT38

Vertex Ordering

Iso- and Other Morphisms

GT35. Maximum Common Subgraph

Instance: Graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$.
Solution: A common subgraph, i.e. subsets ${E_1}' \subseteq E_1$ and ${E_2}' \subseteq E_2$ such that the two subgraphs $G_1' = \langle V_1, {E_1}' \rangle$ and $G_2' = \langle V_2, {E_2}' \rangle$ are isomorphic.
Measure: Cardinality of the common subgraph, i.e., $|E'|$.

Good News: Not harder to approximate than Maximum Clique [Kann, 1992a].
Comment: Transformation to Maximum Clique. Variation in which the degree of the graphs G_1 and G_2 is bounded by the constant B is not harder to approximate than the bounded degree induced common subgraph problem [Kann, 1992a] and is approximable within $B+1$.
Garey and Johnson: GT49

GT36. Maximum Common Induced Subgraph

Instance: Graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$.
Solution: A common induced subgraph, i.e. subsets ${V_1}' \subseteq V_1$ and ${V_2}' \subseteq V_2$ such that $|{V_1}'| = |{V_2}'|$, and the subgraph of G_1 induced by ${V_1}'$ and the subgraph of G_2 induced by ${V_2}'$ are isomorphic.
Measure: Cardinality of the common induced subgraph, i.e., $|{V_1}'|$.

Bad News: Not approximable within $|V|^{\varepsilon}$ for some $\varepsilon > 0$ [Kann, 1992a].

Comment: Transformations to and from MAXIMUM CLIQUE. Variation in which the degree of the graphs G_1 and G_2 is bounded by the constant B is APX-hard and is approximable within $B+1$. If the induced subgraph is restricted to be connected the problem is NPO PB-complete and not approximable within $(|V_1|+|V_2|)^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1992a].

GT37. MAXIMUM COMMON EMBEDDED SUB-TREE

INSTANCE: Trees T_1 and T_2 with labels on the nodes.
SOLUTION: A common embedded sub-tree, i.e. a labeled tree T' that can be embedded into both T_1 and T_2. An embedding from T' to T is an injective function from the nodes of T' to the nodes of T that preserves labels and ancestorship. Note that since fathership does not need to be preserved, T' does not need to be an ordinary subtree.
MEASURE: Cardinality of the common embedded sub-tree, i.e., $|T'|$.

Bad News: APX-hard [Zhang and Jiang, 1994].
Comment: Transformation from MAXIMUM k-SET PACKING. Variation in which the problem is to minimize the edit distance between the two trees is also APX-hard.

Miscellaneous

GT38. LONGEST PATH WITH FORBIDDEN PAIRS

INSTANCE: Graph $G = \langle V, E\rangle$ and a collection $C = \{\langle a_1, b_1\rangle, \ldots, \langle a_m, b_m\rangle\}$ of pairs of vertices from V.
SOLUTION: A simple path in G that contains at most one vertex from each pair in C.
MEASURE: Length of the path, i.e., the number of edges in the path.

Bad News: NPO PB-complete [Berman and Schnitger, 1992].
Comment: Transformation from LONGEST COMPUTATION.
Garey and Johnson: GT54

GT39. SHORTEST PATH WITH FORBIDDEN PAIRS

INSTANCE: Graph $G = \langle V, E\rangle$, a collection $C = \{(a_1, b_1), \ldots, (a_m, b_m)\}$ of pairs of vertices from V, an initial vertex $s \in V$, and a final vertex $f \in V$.
SOLUTION: A simple path from s to f in G that contains at most one vertex from each pair in C.
MEASURE: Length of the path, i.e., the number of edges in the path.

Bad News: NPO PB-complete [Kann, 1993].
Comment: Transformation from SHORTEST COMPUTATION.

GT40. MINIMUM POINT-TO-POINT CONNECTION

INSTANCE: Graph $G = \langle V, E\rangle$, a weight function $w : E \to N$, a set $S = \{s_1, \ldots, s_p\}$ of sources, and a set $D = \{d_1, \ldots, d_p\}$ of destinations.

SOLUTION: A point-to-point connection, i.e., a subset $E' \subseteq E$ such that each source-destination pair is connected in E'.
MEASURE: The weight of the connection, i.e., $\sum_{e \in E'} w(e)$.
Good News: Approximable within $2 - 1/p$ [Goemans and Williamson, 1992].

Network Design

Spanning Trees

ND1. MINIMUM k-SPANNING TREE

INSTANCE: Graph $G = \langle V, E \rangle$, an integer $k \leq n$, and a weight function $w : E \to N$.
SOLUTION: A k-spanning tree, i.e., a subtree T of G of at least k nodes.
MEASURE: The weight of the tree, i.e., $\sum_{e \in T} w(e)$.
Good News: Approximable within $3\sqrt{k}$ [Ravi, Sundaram, Marathe, Rosenkrantz, and Ravi, 1994].
Comment: The restriction to points in the Euclidean plane is approximable within $O(\log k)$ [Garg and Hochbaum, 1994]. The analogous diameter and communication-cost k-spanning tree problems are not in APX [Ravi, Sundaram, Marathe, Rosenkrantz, and Ravi, 1994].

ND2. MINIMUM DEGREE SPANNING TREE

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A spanning tree for G.
MEASURE: The maximum degree of the spanning graph.

Good News: Approximable with an absolute error guarantee of 1 [Fürer and Raghavachari, 1992].
Garey and Johnson: ND1

ND3. MINIMUM WEIGHTED 3-DEGREE SPANNING TREE

INSTANCE: Graph $G = \langle V, E \rangle$ and a weight function $w : E \to N$.
SOLUTION: A spanning tree T for G in which no vertex has degree larger than 3.
MEASURE: The weight of the spanning tree, i.e., $\sum_{(u,v) \in T} w(u, v)$.
Good News: Approximable within 3/2 [Khuller, Raghavachari, and Young, 1994a]
Comment: The 4-degree spanning tree problem is approximable within 5/4. The 5-degree problem is polynomial-time solvable.

ND4. MINIMUM STEINER TREE

INSTANCE: Complete graph $G = \langle V, E \rangle$, edge weights $s : E \to N$ and a subset $S \subseteq V$ of required vertices.
SOLUTION: A Steiner tree, i.e., a subtree of G that includes all the vertices in S.
MEASURE: The sum of the weights of the edges in the subtree.

Good News: Approximable within 16/9 [Berman and Ramaiyer, 1992].

Bad News: Apx-complete [Bern and Plassmann, 1989].

Comment: Variation in which the weights are only 1 or 2 is still Apx-complete, but approximable within 4/3 [Bern and Plassmann, 1989]. When all weights lie in an interval $[\alpha, \alpha(1+1/k)]$ the problem is approximable within $1 + 1/ek + O(1/k^2)$ [Halldórsson, Ueno, Nakao, and Kajitani, 1992]. A prize-collecting variation in which a penalty is associated to each vertex and the goal is to minimize the cost of the tree and the vertices in S not in the tree is approximable within $2 - 1/(|V| - 1)$ [Goemans and Williamson, 1992]. The variation in which an integer $k \leq |S|$ is given in input and at least k vertices of S must be included in the subtree is approximable within $6\sqrt{k}$ [Ravi, Sundaram, Marathe, Rosenkrantz, and Ravi, 1994]. Variation in which there are groups of required vertices and each group must be touched by the Steiner tree is approximable within $g - 1$, where g is the number of groups [Ihler, 1991]. The constrained variation in which the input is extended with a positive integer k and a subset T of E, and the problem is to find the Steiner tree of weight at most k that contains the largest number of edges from T, is not approximable within $|E|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993]. If the solution is allowed to be a forest with at most q trees, for a given constant q, the problem is approximable within $2(1 - 1/(|S| - q + 1))$ [Ravi, 1994].

Garey and Johnson: ND12

ND5. Minimum Geometric Steiner Tree

Instance: Set $P \subseteq Z \times Z$ of points in the plane.

Solution: A finite set of Steiner points, i.e., $Q \subseteq Z \times Z$.

Measure: The total weight of the minimum spanning tree for the vertex set $P \cup Q$, where the weight of an edge $\langle (x_1, y_1), (x_2, y_2) \rangle$ is the discretized Euclidean length

$$\left\lceil \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \right\rceil .$$

Good News: Approximable within $\frac{2}{\sqrt{3}} - \varepsilon$ for some $\varepsilon > 0$ [Du, Zhang, and Feng, 1991].

Comment: If the rectilinear metric $|\langle (x_1, y_1), (x_2, y_2) \rangle| = |x_1 - x_2| + |y_1 - y_2|$ is used the problem admits a PTAS [Berman and Ramaiyer, 1992]. Variation of the rectilinear metric problem in which there are groups of required vertices and each group must be touched by the Steiner tree is Apx-hard, even if the groups are defined by non-overlapping intervals on one of two parallel lines [Ihler, 1991].

Garey and Johnson: ND13

ND6. Minimum Generalized Steiner Network

Instance: Graph $G = \langle V, E \rangle$, a weigh function $w : E \to N$, a capacity function $c : E \to N$, and a requirement function $r : V \times V \to N$.

Solution: A Steiner network over G that satisfies all the requirements and obeys all the capacities, i.e., a function $f : E \to N$ such that, for each edge e, $f(e) \leq c(e)$ and, for any pair of nodes i and j, the number of edge-disjoint paths between i and j is at least $r(i, j)$ where, for each edge e, $f(e)$ copies of e are available.

Measure: The cost of the network, i.e., $\sum_{e \in E} w(e) f(e)$.

Good News: Approximable within $2 \cdot \mathcal{H}(R)$ where R is the maximum requirement and, for any n, $\mathcal{H}(n) = \sum_{i=1}^{n} \frac{1}{i}$ [Goemans, Goldberg, Plotkin, Shmoys, Tardos, and Williamson, 1994].

Comment: When all the requirements are equal, it is approximable within 2 [Khuller and Vishkin, 1994]. The variation in which there are no capacity constraints on the edges is approximable within $2 \cdot \mathcal{H}(|V|)$ [Aggarwal and Garg, 1994].

ND7. MINIMUM ROUTING TREE CONGESTION

INSTANCE: Graph $G = \langle V, E \rangle$ and a weight function $w : E \to N$.

SOLUTION: A routing tree T for G, i.e., a tree T in which each internal vertex has degree 3 and the leaves correspond to vertices of G.

MEASURE: The congestion of the routing tree, i.e., the maximum, for any edge e, of

$$\sum_{(u,v) \in E, u \in S, v \notin S} w(u,v)$$

where S is one of the two connected components obtained by deleting e from T.

Good News: Approximable within $\log n$ [Khuller, Raghavachari, and Young, 1993].

Comment: The algorithm extends to the case when the routing tree is allowed to have vertices of higher degree. If T is required to be a spanning tree and G is complete, the problem is solvable in polynomial time.

ND8. MINIMUM BICONNECTED SPANNING SUBGRAPH

INSTANCE: Graph $G = \langle V, E \rangle$.

SOLUTION: A biconnected spanning subgraph $G' = \langle V, E' \rangle$ for G (connectivity refers to both edge and vertex connectivity).

MEASURE: The cardinality of the spanning subgraph, i.e., $|E'|$.

Good News: Approximable within 5/4 for edge connectivity and within 3/2 for vertex connectivity [Garg, Santosh, and Singla, 1993].

Comment: Variation in which each edge has a nonnegative weight and the objective is to minimize the total weight of the spanning subgraph is approximable within 2. [Khuller and Vishkin, 1994].

Cuts and Connectivity

ND9. MAXIMUM CUT

INSTANCE: Graph $G = \langle V, E \rangle$.

SOLUTION: A partition of V into disjoint sets V_1 and V_2.

MEASURE: The cardinality of the cut, i.e., the number of edges with one end point in V_1 and one endpoint in V_2.

Good News: Approximable within 1.14 [Goemans and Williamson, 1994].

Bad News: APX-complete [Papadimitriou and Yannakakis, 1991].

Comment: Transformation from MAXIMUM NOT-ALL-EQUAL 3-SATISFIABILITY. Variation in which the degree of G is bounded by a constant B is still APX-complete [Papadimitriou and Yannakakis, 1991]. The weighted problem, where every edge is assigned a nonnegative weight and the objective is to maximize the total weight of the edges in the cut is also approximable within 1.14 [Goemans and Williamson, 1994].

Garey and Johnson: ND16

ND10. Maximum Directed Cut

Instance: Directed graph $G = \langle V, A\rangle$.
Solution: A partition of V into disjoint sets V_1 and V_2.
Measure: The cardinality of the cut, i.e., the number of arcs with one end point in V_1 and one endpoint in V_2.

Good News: Approximable within 1.26 [Goemans and Williamson, 1994].
Bad News: Apx-complete [Papadimitriou and Yannakakis, 1991].
Comment: The weighted problem, where every arc is assigned a nonnegative weight and the objective is to maximize the total weight of the arcs in the cut is also approximable within 1.26 [Goemans and Williamson, 1994].

ND11. Maximum k-Cut

Instance: Graph $G = \langle V, E\rangle$, a weight function $w : E \to N$, and an integer $k \in [2..|V|]$.
Solution: A partition of V into k disjoint sets $F = \{C_1, C_2, \ldots, C_k\}$.
Measure: The sum of the weight of the edges between the disjoint sets, i.e.,

$$\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{\substack{v_1 \in C_i \\ v_2 \in C_j}} w(\{v_1, v_2\}).$$

Good News: Approximable within k [Sahni and Gonzalez, 1976].
Bad News: Apx-complete.
Comment: The constrained variation in which the input is extended with a positive integer W, a vertex $v_0 \in V$ and a subset S of V, and the problem is to find the 2-cut of weight at least W with the largest number of vertices from S on the same side as v_0, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].

ND12. Minimum Network Inhibition on Planar Graphs

Instance: Planar graph $G = \langle V, E\rangle$, capacity function $c : E \to N$, destruction cost function $d : E \to N$, and budget B.
Solution: An attack strategy to the network, i.e., a function $\alpha : E \to [0, 1]$ such that $\sum_{e \in E} \alpha(e)d(e) \leq B$.
Measure: The capability left in the damaged network, i.e., the minimum cut in G with capacity c' defined as $c'(e) = \alpha(e)c(e)$.
Good News: Admits an FPTAS [Phillips, 1993].

ND13. Minimum k-Cut

Instance: Graph $G = \langle V, E\rangle$, a weight function $w : E \to N$, and an integer $k \in [2..|V|]$.
Solution: A partition of V into k disjoint sets $F = \{C_1, C_2, \ldots, C_k\}$.

MEASURE: The sum of the weight of the edges between the disjoint sets, i.e.,

$$\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{\substack{v_1 \in C_i \\ v_2 \in C_j}} w(\{v_1, v_2\}).$$

Good News: Approximable within $2 - \frac{2}{k}$ [Saran and Vazirani, 1991].

Comment: Solvable in polynomial time $O(|V|^{k^2})$ for fixed k [Goldschmidt and Hochbaum, 1988]. If the sets in the partition are restricted to be of equal size, the problem is approximable within $|V| \cdot (k-1)/k$ [Saran and Vazirani, 1991].

ND14. MINIMUM VERTEX k-CUT

INSTANCE: Graph $G = \langle V, E \rangle$, a set $S = \{s_1, t_1, \ldots, s_k, t_k\}$ of special vertices, and a weight function $w : V - S \rightarrow N$, and an integer k.

SOLUTION: A vertex k-cut, i.e., a subset $C \subseteq V - S$ of vertices such that their deletion from G disconnects each s_i from t_i for $1 \leq i \leq k$.

MEASURE: The sum of the weight of the vertices in the cut, i.e., $\sum_{v \in C} w(v)$.

Good News: Approximable within $O(\log |V|)$ [Garg, Vazirani, and Yannakakis, 1994].

ND15. MINIMUM MULTIWAY CUT

INSTANCE: A graph $G = \langle V, E \rangle$, a set $S \subseteq V$ of terminals, and a weight function $w : E \rightarrow N$.

SOLUTION: A multiway cut, i.e., a set $E' \subseteq E$ such that the removal of E' from E disconnects each terminal from all the others.

MEASURE: The weight of the cut, i.e., $\sum_{e \in E'} w(e)$.

Good News: Approximable within $2 - \frac{2}{|S|}$ [Dahlhaus, Johnson, Papadimitriou, Seymour, and Yannakakis, 1992].

Bad News: APX-complete [Dahlhaus, Johnson, Papadimitriou, Seymour, and Yannakakis, 1992].

Comment: It remains APX-complete even if $|S| \geq 3$ is fixed. For $|S| = 4$ and $|S| = 8$ it is approximable within 4/3 and 12/7, respectively. In the case of directed graphs the problem is approximable within $O(\log |S|)$ and APX-hard [Garg, Vazirani, and Yannakakis, 1994]. The vertex deletion variation is approximable within $2 - 2/|S|$ and is APX-complete [Garg, Vazirani, and Yannakakis, 1994]. If S is formed by pairs of vertices s_i and t_i and we require to disconnect only these pairs, the problem is approximable within $O(\log |S|)$ both in the case of edge deletion [Garg, Vazirani, and Yannakakis, 1993b] and in the case of vertex deletion [Garg, Vazirani, and Yannakakis, 1994]. It is APX-complete and approximable within 2 for trees of height one and unit edge weight.

ND16. MINIMUM RATIO-CUT

INSTANCE: Graph $G = \langle V, E \rangle$.

SOLUTION: A partition of V into disjoint sets V_1 and V_2.

MEASURE: The cardinality of the cut divided by the product of the cardinalities of the disjoint sets, i.e., $c/(|V_1| \cdot |V_2|)$ where c is the number of edges with one end point in V_1 and one endpoint in V_2.

Good News: Approximable within $O(\log |V|)$ [Leighton and Rao, 1988].

ND17. MINIMUM QUOTIENT CUT

INSTANCE: Graph $G = \langle V, E \rangle$, a vertex-weight function $c : V \to N$, and an edge-cost function $w : E \to N$.
SOLUTION: A cut C, i.e., a subsets $C \subseteq V$.
MEASURE: The quotient of the cut, i.e.,

$$\frac{c(C)}{\min\{w(C), w(\overline{C})\}}$$

where $c(C)$ denotes the sum of the costs of the edges (u, v) such that either $u \in C$ and $v \notin C$ or $u \notin C$ and $v \in C$ and, for any subset $V' \subseteq V$, $w(V')$ denotes the sum of the weights of the vertices in V'.

Good News: Approximable within $O(\log |V|)$ [Leighton and Rao, 1988].
Comment: Admits a PTAS for planar graphs [Park and Phillips, 1993].

ND18. MINIMUM BICONNECTIVITY AUGMENTATION

INSTANCE: Graph $G = \langle V, E \rangle$ and a symmetric weight function $w : V \times V \to N$.
SOLUTION: A connectivity augmenting set E' for G, i.e., a set E' of unordered pairs of vertices from V such that $G' = \langle V, E \cup E' \rangle$ is biconnected.
MEASURE: The weight of the augmenting set, i.e., $\sum_{(u,v) \in E'} w(u, v)$.
Good News: Approximable within 2 [Khuller and Thurimella, 1993].
Comment: If the weight function satisfies the triangle inequality, the problem is approximable within 3/2 [Frederickson and Jájá, 1981].

Routing Problems

ND19. TRAVELING SALESPERSON PROBLEM

INSTANCE: Set C of m cities, distances $d(c_i, c_j) \in N$ for each pair of cities $c_i, c_j \in C$.
SOLUTION: A tour of C, i.e., a permutation $\pi : [1..m] \to [1..m]$.
MEASURE: The length of the tour, i.e., $d\left(\{c_{\pi(m)}, c_{\pi(1)}\}\right) + \sum_{i=1}^{m-1} d\left(\{c_{\pi(i)}, c_{\pi(i+1)}\}\right)$.

Bad News: NPO-complete [Orponen and Mannila, 1987].
Garey and Johnson: ND22

ND20. METRIC TRAVELING SALESPERSON PROBLEM

INSTANCE: Set C of m cities, distances $d(c_i, c_j) \in N$ satisfying the triangle inequality.
SOLUTION: A tour of C, i.e., a permutation $\pi : [1..m] \to [1..m]$.
MEASURE: The length of the tour.
Good News: Approximable within 3/2 [Christofides, 1976].
Bad News: APX-complete [Papadimitriou and Yannakakis, 1993].
Comment: Variation in which the distances are only 1 or 2 is still APX-complete, but approximable within 7/6 [Papadimitriou and Yannakakis, 1993]. A prize-collecting variation in which a penalty is associated to each vertex and the goal is to minimize the cost of the tour and the vertices not in the tour is approximable within $2 - 1/(|V| - 1)$ [Goemans and Williamson, 1992].
Garey and Johnson: ND23

ND21. METRIC TRAVELING k-SALESPERSON PROBLEM

INSTANCE: Set C of m cities, an initial city $s \in C$, distances $d(c_i, c_j) \in N$ satisfying the triangle inequality.
SOLUTION: A collection of k subtours, each containing the initial city s, such that each city is in at least one subtour.
MEASURE: The maximum length of the k subtours.
Good News: Approximable within $1 - 1/k$ plus the performance ratio of the METRIC TRAVELING SALESPERSON PROBLEM, i.e., within $\frac{5}{2} - \frac{1}{k}$ [Frederickson, Hecht, and Kim, 1978].

ND22. METRIC BOTTLENECK WANDERING SALESPERSON PROBLEM

INSTANCE: Set C of m cities, an initial city $s \in C$, a final city $f \in C$, distances $d(c_i, c_j) \in N$ satisfying the triangle inequality.
SOLUTION: A simple path from the initial city s to the final city f passing through all cities in C, i.e., a permutation $\pi : [1..m] \to [1..m]$ such that $v_{\pi(1)} = s$ and $v_{\pi(m)} = f$.
MEASURE: The length of the largest distance in the path, i.e.,

$$\max_{i \in [1..m-1]} d\left(\{c_{\pi(i)}, c_{\pi(i+1)}\}\right).$$

Good News: Approximable within 2 [Hochbaum and Shmoys, 1986].
Bad News: Not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].
Comment: The same positive and negative results hold even if X is a set of point in d-dimensional space with the L_1 or L_∞ metric. If the L_2 metric is used then the upper bound is 1.969 [Feder and Greene, 1988]. Similar results hold for the variation in which it is required to minimize the distance between any point in a cluster and a cluster center which can be any point in the space [Feder and Greene, 1988].
Garey and Johnson: ND24

ND23. MINIMUM CHINESE POSTMAN FOR MIXED GRAPHS

INSTANCE: Mixed graph $G = \langle V, A, E \rangle$, length $l(e) \in N$ for each $e \in A \cup E$.

SOLUTION: A cycle in G (possibly containing repeated vertices) that includes each directed and undirected edge at least once, traversing directed edges only in the specified direction.
MEASURE: The total length of the cycle.

Good News: Approximable within 5/3 [Frederickson, 1979].
Comment: Approximable within 3/2 for planar graphs [Frederickson, 1979].
Garey and Johnson: ND25

ND24. MINIMUM k-CHINESE POSTMAN PROBLEM

INSTANCE: Multigraph $G = \langle V, E\rangle$, initial vertex $s \in V$, length $l(e) \in N$ for each $e \in E$.
SOLUTION: A collection of k cycles, each containing the initial vertex s, that collectively traverse every edge in the graph at least once.
MEASURE: The maximum length of the k cycles.

Good News: Approximable within $2 - 1/k$ [Frederickson, Hecht, and Kim, 1978].

ND25. MINIMUM STACKER CRANE PROBLEM

INSTANCE: Mixed graph $G = \langle V, A, E\rangle$, length $l(e) \in N$ for each $e \in A \cup E$ such that for every arc there is a parallel edge of no greater length.
SOLUTION: A cycle in G (possibly containing repeated vertices) that includes each directed edge in A at least once, traversing such edges only in the specified direction.
MEASURE: The total length of the cycle.

Good News: Approximable within 9/5 [Frederickson, Hecht, and Kim, 1978].
Garey and Johnson: ND26

ND26. MINIMUM k-STACKER CRANE PROBLEM

INSTANCE: Mixed graph $G = \langle V, A, E\rangle$, initial vertex $s \in V$, length $l(e) \in N$ for each $e \in A \cup E$,
SOLUTION: A collection of k cycles, each containing the initial vertex s, that collectively traverse each directed edge in A at least once.
MEASURE: The maximum length of the k cycles.

Good News: Approximable within $14/5 - 1/k$ [Frederickson, Hecht, and Kim, 1978].

ND27. MINIMUM GENERAL ROUTING

INSTANCE: Graph $G = \langle V, E\rangle$, length $l(e) \in N$ for each $e \in E$, subset $E' \subseteq E$, subset $V' \subseteq V$.
SOLUTION: A cycle in G that visits each vertex in V' exactly once and traverses each edge in E'.
MEASURE: The total length of the cycle.

Good News: Approximable within 3/2 [Jansen, 1992].
Comment: The special case where $V' = V$ is called the rural postman problem.
Garey and Johnson: Generalization of ND27

ND28. Longest Path

Instance: Graph $G = \langle V, E \rangle$.
Solution: Simple path in G, i.e., a sequence of distinct vertices $v_1, v_2, \ldots, v_m$ such that, for any $1 \le i \le m-1$, $(v_i, v_{i+1}) \in E$.
Measure: Length of the path, i.e., the number of edges in the path.

Bad News: Not in Apx [Karger, Motwani, and Ramkumar, 1993].
Comment: Transformation from Min TSP(1,2): Apx-hard and self-improvable. Not approximable within $2^{\log^{1-\varepsilon}|V|}$ for any $\varepsilon > 0$ unless NP$\subset$QP [Karger, Motwani, and Ramkumar, 1993]. Approximable within $O(|V|/\log|V|)$ for 1-tough graphs, i.e., graphs such that, for any subset $V' \subseteq V$, the induced graph $G = \langle V - V', E \rangle$ has at most $|V'|$ connected components (observe that Hamiltonian graphs are 1-tough). If a polynomial-time algorithm exists with relative error $|V|/(|V| - |V|^{\varepsilon})$ then P = NP. Similar results hold for a chromatic version of the problem [Bellare, 1993].
Garey and Johnson: ND29

ND29. Shortest Weight-Constrained Path

Instance: Graph $G = \langle V, E \rangle$, length function $l : E \to N$, weight function $w : E \to N$, specified vertices $s, t \in V$, and integer W.
Solution: A simple path in G with total weight at most W, i.e., a sequence of distinct vertices $s = v_1, v_2, \ldots, v_m = t$ such that, for any $1 \le i \le m-1$, $(v_i, v_{i+1}) \in E$ and $\sum_{i=1}^{m_1} w(v_i, v_{i+1}) \le W$.
Measure: The length of the path, i.e., $\sum_{i=1}^{m_1} l(v_i, v_{i+1})$.
Good News: Admits an FPTAS [Phillips, 1993].

ND30. Minimum Rectilinear Global Routing

Instance: $m \times n$-array of gates, collection C of nets, i.e., 3-sets of gates.
Solution: Wires following rectilinear paths connecting the gates in each net.
Measure: The largest number of wires in the same channel between two gates in the array.

Good News: Admits a PTAS$^{\infty}$ if $opt \in \omega(\ln(mn))$ [Raghavan and Thompson, 1991].
Comment: Approximable within $1 + (e-1)\sqrt{2\ln(mn)/opt}$ if $opt > 2\ln(mn)$. In Apx if $opt \in \Omega(\ln(mn))$. The approximation algorithm will work also for nets with more than three gates, but the running time is exponential in the number of terminals.

Flow Problems

ND31. Maximum k-Multicommodity Flow

Instance: Graph $G = \langle V, E \rangle$, edge capacities $u : E \to Z^+$, a set of k commodities $C = \{\langle s_1, t_1, d_1 \rangle, \ldots, \langle s_k, t_k, d_k \rangle\}$ where $s_i \in V$ specifies the source, $t_i \in V$ the sink, and $d_i \in Z^+$ the demand for each commodity.
Solution: The flow of each commodity through each edge in E.

MEASURE: The proportion of the demand of the flow of the commodity that has the smallest proportion, i.e.,

$$\min_{1 \le i \le k} \frac{\text{the flow of the commodity } (s_i, t_i, d_i) \text{ from } s \text{ to } t}{d_i}.$$

Good News: Admits an FPTAS [Klein, Agrawal, Ravi, and Rao, 1990].

Comment: Approximable within $1+\varepsilon$ in time $O\left((|E||V|\log^3|V|k^2\log k)/\varepsilon^2\right)$ for each $\varepsilon > 0$ [Leighton, Makedon, Plotkin, Stein, Tardos, and Tragoudas, 1991].

ND32. MAXIMUM INTEGRAL k-MULTICOMMODITY FLOW ON TREES

INSTANCE: A tree $T = \langle V, E\rangle$, a capacity function $c : E \to N$ and k pairs of vertices (s_i, t_i).

SOLUTION: A flow f_i for each pair (s_i, t_i) with $f_i \in N$ such that, for each $e \in E$, $\sum_{i=1}^k f_i q_i(e) \le c(e)$ where $q_i(e) = 1$ if e belongs to the unique path from s_i and t_i, 0 otherwise.

MEASURE: The sum of the flows, i.e., $\sum_{i=1}^k f_i$.

Good News: Approximable within 2 [Garg, Vazirani, and Yannakakis, 1993b].

Bad News: APX-complete [Garg, Vazirani, and Yannakakis, 1993b].

Comment: Transformation from MAXIMUM 3-DIMENSIONAL MATCHING. It remains APX-complete even if the edge capacities are 1 and 2.

ND33. MAXIMUM PRIORITY FLOW

INSTANCE: Directed graph $G = \langle V, E\rangle$, sources $s_1, \ldots, s_k \in V$, sinks $t_1, \ldots, t_k \in V$, a capacity function $c : E \to R$, a bound function $b : V \to R$, and, for any vertex v, a partial order on the set of edges leaving v.

SOLUTION: A priority flow f, i.e., a function $f : E \to R$ such that (a) for any edge e, $f(e) \le c(e)$, (b) for any vertex $v \in V - \{s_1, \ldots, s_k, t_1, \ldots, t_k\}$, the flow is conserved at v, (c) for any vertex v, the flow leaving v is at most $b(v)$, and (d) for any vertex v and for any pair of edges e_1, e_2 leaving v, if $f(e_1) < c(e_1)$ and e_1 is less than e_2, then $f(e_2) = 0$.

MEASURE: The amount of flow entering sink t_1, i.e., $\sum_{(x,t_1)\in E} f(x, t_1)$.

Bad News: Not approximable within $2^{\theta(\log^\varepsilon n)}$ for any ε unless NP $\subseteq$ DTIME$(n^{d\log^{1/\varepsilon} n})$ [Bellare, 1993].

Comment: Does not admit a PTAS.

Miscellaneous

ND34. MINIMUM k-CENTER

INSTANCE: Complete graph $G = \langle V, E\rangle$ and distances $d(v_i, v_j) \in N$ satisfying the triangle inequality.

SOLUTION: A k-center set, i.e., a subset $C \subseteq V$ with $|C| = k$.

MEASURE: The maximum distance from a vertex to its nearest center, i.e., $\max_{v\in V}\min_{c\in C} d(v, c)$.

Good News: Approximable within 2 [Hochbaum and Shmoys, 1986].

Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].

Comment: Variation in which the number of vertices each center can serve is bounded by a constant L, is approximable within 10 [Bar-Ilan and Peleg, 1991]. The converse problem, where the maximum distance from each vertex to its center is given and the number of centers is to be minimized, is approximable within $\log L + 1$ [Bar-Ilan and Peleg, 1991]. The rectilinear k-center problem, where the vertices lie in the plane and the rectilinear metric is used, is approximable within 2, but is not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Ko, Lee, and Chang, 1990].

Garey and Johnson: Similar to ND50

ND35. MINIMUM k-SUPPLIER

INSTANCE: Complete graph $G = \langle V_C \cup V_S, E\rangle$ and distances $d(v_i, v_j) \in N$ satisfying the triangle inequality.

SOLUTION: A k-supplier set, i.e., a subset $S \subseteq V_S$ with $|S| = k$.

MEASURE: The maximum distance from a customer vertex to its nearest supplier, i.e., $\max_{v \in V_C} \min_{s \in S} d(v, s)$.

Good News: Approximable within 3 [Hochbaum and Shmoys, 1986].

Bad News: Not approximable within $3 - \varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].

ND36. MINIMUM k-MEDIAN

INSTANCE: Graph $G = \langle V, E\rangle$ and length function $l : E \to N$.

SOLUTION: A k-median set, i.e., a subset $V' \subseteq V$ with $|V'| = k$.

MEASURE: The sum of the distances from each vertex to its nearest median, i.e.,

$$\sum_{v \in V} d(v)$$

where $d(v)$ is the length of the shortest path from v to the closest vertex in V'.

Bad News: Not in APX [Lin and Vitter, 1992].

Comment: The problem is in APX if a small violation of the cardinality of the median set is allowed.

Garey and Johnson: ND51

ND37. MAXIMUM k-FACILITY DISPERSION

INSTANCE: Complete graph $G = \langle V, E\rangle$ and distances $d(v_i, v_j) \in N$ satisfying the triangle inequality.

SOLUTION: A set of k facilities, i.e., a subset $F \subseteq V$ with $|F| = k$.

MEASURE: The minimum distance between two facilities, i.e., $\min_{f_1, f_2 \in F} d(f_1, f_2)$.

Good News: Approximable within 2 [Ravi, Rosenkrantz, and Tayi, 1991].

Bad News: Not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Ravi, Rosenkrantz, and Tayi, 1991].

Comment: Variation in which the measure is the average distance between any pair of facilities is approximable within 4 [Ravi, Rosenkrantz, and Tayi, 1991].

ND38. MINIMUM k-SWITCHING NETWORK

INSTANCE: Complete graph $G = \langle V, E \rangle$ and distances $d(v_i, v_j) \in N$ satisfying the triangle inequality.
SOLUTION: A partition $F = \{A_1, A_2, \ldots, A_k, B_1, B_2, \ldots, B_k\}$ of V.
MEASURE: Maximum distance between vertices in different sets with the same index, i.e.,

$$\max_{i \in [1..k]} \max_{\substack{v_1 \in A_i \\ v_2 \in B_i}} d(v_1, v_2).$$

Good News: Approximable within 3 [Hochbaum and Shmoys, 1986].
Bad News: Not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].

ND39. MINIMUM BEND NUMBER

INSTANCE: Directed planar graph $G = \langle V, E \rangle$.
SOLUTION: A planar orthogonal drawing of G, i.e., a drawing mapping vertices of G into points in the plane and edges of G into chains of horizontal and vertical segments such that no two edges cross.
MEASURE: Number of bends in the drawing.
Bad News: Not approximable within $1 + \frac{|V|^{1-\varepsilon}}{opt(G)}$ for any $\varepsilon > 0$ [Garg and Tamassia, 1994].

ND40. MINIMUM LENGTH TRIANGULATION

INSTANCE: Collection $C = \{(a_i, b_i) : 1 \le i \le n\}$ of pairs of integers giving the coordinates of n points in the plane.
SOLUTION: A triangulation of the set of points represented by C, i.e., a collection E of non-intersecting line segments each joining two points in C that divides the interior of the convex hull into triangular regions.
MEASURE: The discrete-Euclidean length of the triangulation, i.e.,

$$\left\lceil \sum_{((a_i,b_i),(a_j,b_j)) \in E} \sqrt{(a_i - a_j)^2 + (b_i - b_j)^2} \right\rceil.$$

Good News: Approximable within $O(\log n)$ [Clarkson, 1991].
Comment: Note that the problem is not known to be NP-complete. The Steiner variation in which the point set of E must be a superset of C is approximable within 'a number in the hundreds' [Eppstein, 1992].
Garey and Johnson: OPEN12

ND41. MINIMUM SEPARATING SUBDIVISION

INSTANCE: A family of disjoint polygons $P_1, \ldots, P_k$.
SOLUTION: A separating subdivision, i.e., a family of k polygons $R_1, \ldots, R_k$ with pairwise disjoint boundaries such that, for each i, $P_i \subseteq R_i$.
MEASURE: The size of the subdivision, i.e., the total number of edges of the polygons $R_1, \ldots, R_k$.

Good News: Approximable within 7 [Mitchell and Suri, 1992].

Comment: The problem of separating a family of three-dimensional convex polyhedra is approximable within $O(\log n)$ while the problem of separating two d-dimensional convex polyhedra is approximable within $O(d \log n)$ where n denotes the number of facets in the input family.

Sets and Partitions

Covering, Hitting, and Splitting

SP1. Maximum 3-Dimensional Matching

Instance: Set $T \subseteq X \times Y \times Z$, where X, Y, and Z are disjoint.

Solution: A matching for T, i.e., a subset $M \subseteq T$ such that no elements in M agree in any coordinate.

Measure: Cardinality of the matching, i.e., $|M|$.

Good News: Approximable within 2 [Halldórsson, 1994].

Bad News: Apx-complete [Kann, 1991].

Comment: Transformation from Maximum 3-Satisfiability. Admits a PTAS for 'planar' instances [Nishizeki and Chiba, 1988]. Variation in which the number of occurrences of any element in X, Y or Z is bounded by a constant B is Apx-complete [Kann, 1991]. The generalized Maximum k-Dimensional Matching problem is approximable within $(k+1)/2$ [Halldórsson, 1994]. The constrained variation in which the input is extended with a subset S of T, and the problem is to find the 3-dimensional matching that contains the largest number of elements from S, is not approximable within $|T|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].

Garey and Johnson: SP1

SP2. Maximum Set Packing

Instance: Collection C of finite sets.

Solution: A set packing, i.e., a collection of disjoint sets $C' \subseteq C$.

Measure: Cardinality of the set packing, i.e., $|C'|$.

Bad News: Not approximable within $|C|^\varepsilon$ for some $\varepsilon > 0$ [Arora, Lund, Motwani, Sudan, and Szegedy, 1992].

Comment: Equivalent to Maximum Clique under PTAS-reduction [Ausiello, D'Atri, and Protasi, 1980]. The problem Maximum k-Set Packing, the variation in which the cardinality of all sets in C are bounded from above by a constant $k \geq 3$, is Apx-complete [Kann, 1991], and is approximable within $(k+1)/2$ [Halldórsson, 1994]. It is still Apx-complete when the number of occurrences in C of any element is bounded by a constant $B \geq 3$.

Garey and Johnson: SP3

SP3. Maximum Set Splitting

Instance: Collection C of subsets of a finite set S.

SOLUTION: A partition of S into two disjoint subsets S_1 and S_2.
MEASURE: Cardinality of the subsets in C that are not entirely contained in either S_1 or S_2.

Bad News: APX-hard [Petrank, 1993].
Comment: Transformation from MAXIMUM NOT-ALL-EQUAL 3-SATISFIABILITY.
Garey and Johnson: SP4

SP4. MINIMUM SET COVER

INSTANCE: Collection C of subsets of a finite set S.
SOLUTION: A set cover for S, i.e., a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C'.
MEASURE: Cardinality of the set cover, i.e., $|C'|$.

Good News: Approximable within $1 + \ln |S|$ [Johnson, 1974].
Bad News: Not approximable within $c \log_2 |S|$ for any $c < 1/4$, unless NP$\subset$QP [Lund and Yannakakis, 1993a].
Comment: Not approximable within $c \log_2 |S|$ for any $c < 1/8$, unless NP$\subset$ DTIME $\left(|S|^{\log\log|S|}\right)$ [Bellare, Goldwasser, Lund, and Russell, 1993]. Equivalent to MINIMUM DOMINATING SET under L-reduction and equivalent to MINIMUM HITTING SET [Ausiello, D'Atri, and Protasi, 1980]. The above nonapproximability results are still true for an exact cover, i.e., if the sets in the set cover are restricted to be disjoint. The problem MINIMUM k-SET COVER, the variation in which the cardinality of all sets in C are bounded from above by a constant k is APX-complete and is approximable within $\sum_{i=1}^{k} \frac{1}{i}$ [Johnson, 1974]. It is still APX-complete when the number of occurrences in C of any element is bounded by a constant. It is approximable within 2 if the set system (S, C) is tree representable [Garg, Vazirani, and Yannakakis, 1993b]. The constrained variation in which the input is extended with a positive integer k and a subset T of C, and the problem is to find the set cover of size k that contains the largest number of subsets from T, is not approximable within $|C|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: SP5

SP5. MINIMUM EXACT COVER

INSTANCE: Collection C of subsets of a finite set S.
SOLUTION: A set cover for S, i.e., a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C'.
MEASURE: Sum of cardinalities of the subsets in the set cover, i.e., $\sum_{c \in C'} |c|$.

Good News: Approximable within $1 + \ln |S|$ [Johnson, 1974].
Bad News: Not approximable within $c \log_2 |S|$ for any $c < 1/4$ unless NP$\subset$QP [Lund and Yannakakis, 1993a].
Comment: Not approximable within $c \log_2 |S|$ for any $c < 1/8$, unless NP$\subset$ DTIME $\left(|S|^{\log\log|S|}\right)$ [Bellare, Goldwasser, Lund, and Russell, 1993]. The only difference between MINIMUM SET COVER and MINIMUM EXACT COVER is the definition of the objective function. Transformation from MINIMUM SET COVER [Lund and Yannakakis, 1993a].

SP6. MINIMUM TEST COLLECTION

INSTANCE: Collection C of subsets of a finite set S.
SOLUTION: A subcollection $C' \subseteq C$ such that for each pair of distinct elements $x_1, x_2 \in S$ there is some set $c \in C'$ that contains exactly one of x_1 and x_2.
MEASURE: Cardinality of the subcollection, i.e., $|C'|$.

Good News: Approximable within $1 + 2\ln|S|$.
Comment: Transformation to MINIMUM SET COVER [Kann, 1992b]. Observe that every solution has cardinality at least $\lceil \log |S| \rceil$.
Garey and Johnson: SP6

SP7. MINIMUM HITTING SET

INSTANCE: Collection C of subsets of a finite set S.
SOLUTION: A hitting set for C, i.e., a subset $S' \subseteq S$ such that S' contains at least one element from each subset in C.
MEASURE: Cardinality of the hitting set, i.e., $|C'|$.

Good News: Approximable within $1 + \ln|S|$.
Bad News: Not approximable within $c \log_2 |S|$ for any $c < 1/4$ unless NP$\subset$QP [Lund and Yannakakis, 1993a].
Comment: Not approximable within $c \log_2 |S|$ for any $c < 1/8$, unless NP$\subset$ DTIME$\left(|S|^{\log\log|S|}\right)$ [Bellare, Goldwasser, Lund, and Russell, 1993]. The constrained variation in which the input is extended with a subset T of S, and the problem is to find the hitting set that contains the largest number of elements from T, is not approximable within $|S|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: SP8

Weighted Set Problems

SP8. MAXIMUM CONSTRAINED PARTITION

INSTANCE: Finite set A and a size $s(a) \in Z^+$ for each $a \in A$, element $a_0 \in A$, and a subset $S \subseteq A$.
SOLUTION: A partition of A, i.e., a subset $A' \subseteq A$ such that $\sum_{a\in A'} s(a) = \sum_{a \in A-A'} s(a)$.
MEASURE: Number of elements from S on the same side of the partition as a_0.

Bad News: Not approximable within $|A|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: Similar to SP12

SP9. MINIMUM 3-DIMENSIONAL ASSIGNMENT

INSTANCE: Three sets X, Y, and W and a cost function $c : X \times Y \times W \to N$.
SOLUTION: An assignment A, i.e., a subset $A \subseteq X \times Y \times W$ such that every element of $X \cup Y \cup W$ belongs to exactly one triple in A.
MEASURE: The cost of the assignment, i.e., $\sum_{(x,y,w)\in A} c(x, y, w)$.

Bad News: Not in APX [Crama and Spieksma, 1992].
Comment: The negative result holds even if c is either defined as $c(x, y, w) = d(x, y) + d(x, w) + d(y, w)$ or defined as $c(x, y, w) = \min\{d(x, y) + d(x, w), d(x, y) + d(y, w), d(x, w) + d(y, w)\}$ where d is any distance function. In these cases, however, the problem is approximable within 4/3 if d satisfies the triangle inequality. Similar results hold for the k-dimensional problem [Bandelt, Crama, and Spieksma, 1991].
Garey and Johnson: Weighted version of SP2

SP10. MAXIMUM CAPACITY REPRESENTATIVES

INSTANCE: Disjoint sets $S_1, \ldots, S_m$ and, for any $i \neq j$, $x \in S_i$, and $y \in S - j$, a nonnegative capacity $c(x, y)$.
SOLUTION: A system of representatives T, i.e., a set T such that, for any i, $|T \cap S_i| = 1$.
MEASURE: The capacity of the system of representatives, i.e., $\sum_{x,y \in T} c(x, y)$.
Bad News: Not approximable within $2^{\theta(\log^{\varepsilon} n)}$ for any ε unless NP $\subseteq$ DTIME$(n^{d \log^{1/\varepsilon} n})$ [Bellare, 1993].
Comment: Does not admit a PTAS.

Storage and Retrieval

Data Storage

SR1. MINIMUM BIN PACKING

INSTANCE: Finite set U of items, a size $s(u) \in Z^+$ for each $u \in U$, and a positive integer bin capacity B.
SOLUTION: A partition of U into disjoint sets $U_1, U_2, \ldots, U_m$ such that the sum of the a partition of U such that the sum of the items in each U_i is B or less.
MEASURE: The number of used bins, i.e., the number of disjoint sets, m.

Good News: Approximable within 11/9 [Johnson, Demers, Ullman, Garey, and Graham, 1974].
Bad News: Not approximable within 3/2.
Comment: Admits an FPTAS$^{\infty}$, that is, is approximable within $1 + \varepsilon$ in time polynomial in $1/\varepsilon$, where $\varepsilon = O(\log^2(opt)/opt)$ [Karmarkar and Karp, 1982]. Not APX-complete, but it is NP-complete to decide whether two bins are enough. A survey of approximation algorithms for MINIMUM BIN PACKING is found in [Coffman, Garey, and Johnson, 1984]. If a partial order on U is defined and we require the bin packing to obey this order, then the problem is approximable within 2 [Wee and Magazine, 1982], and is not in FPTAS$^{\infty}$ [Queyranne, 1985].
Garey and Johnson: SR1

SR2. MINIMUM HEIGHT THREE DIMENSIONAL PACKING

INSTANCE: Set of boxes $B = \{(x_i, y_i, z_i)\}$ with positive integer sizes (width x_i, depth y_i and height z_i), a large box with positive integer sizes width w, depth d and infinite height.
SOLUTION: A packing P of the boxes B in the large box. The boxes must be packed orthogonally and oriented.

MEASURE: Height of the packing P.

Good News: Approximable within 3.25 [Li and Cheng, 1990].

Comment: The two-dimensional variation in which a set of rectangles of dimensions bounded below by a constant, is to be packed into a strip of width 1 admits a PTAS [Fernandez de la Vega and Zissimopoulos, 1991]. There are lots of variants of packing problems. A survey of approximation results of packing problems can be found in [Coffman, Garey, and Johnson, 1984].

Compression and Representation

SR3. SHORTEST COMMON SUPERSEQUENCE

INSTANCE: Finite alphabet Σ, finite set R of strings from Σ^*.

SOLUTION: A string $w \in \Sigma^*$ such that each string $x \in R$ is a subsequence of w, i.e. one can get x by taking away letters from w.

MEASURE: Length of the supersequence, i.e., $|w|$.

Bad News: Not in APX [Jiang and Li, 1994].

Comment: Transformation from MINIMUM FEEDBACK VERTEX SET and self-improvability. Not approximable within $\log^\delta |R|$ for a given $\delta > 0$ unless NP $\subset$ DTIME$\left(n^{\text{polylog} n}\right)$ [Jiang and Li, 1994]. APX-complete if the size of the alphabet Σ is fixed [Jiang and Li, 1994] and [Bonizzoni, Duella, and Mauri, 1994].

Garey and Johnson: SR8

SR4. SHORTEST COMMON SUPERSTRING

INSTANCE: Finite alphabet Σ, finite set R of strings from Σ^*.

SOLUTION: A string $w \in \Sigma^*$ such that each string $x \in R$ is a substring of w, i.e. $w = w_0 x w_1$ where $w_0, w_1 \in \Sigma^*$.

MEASURE: Length of the superstring, i.e., $|w|$.

Good News: Approximable within 2.89 [Teng and Yai, 1993].

Bad News: APX-complete [Blum, Jiang, Li, Tromp, and Yannakakis, 1991].

Comment: Transformation from METRIC TRAVELING SALESPERSON PROBLEM with distances one and two. Variation in which there are negative strings in the input and a solution cannot contain any negative string as a substring, is approximable within $O(\log opt)$ [Li, 1990]. If the number of negative strings is constant, or if no negative strings contain positive strings as substrings, the problem is approximable within some constant [Jiang and Li, 1993].

Garey and Johnson: SR9

SR5. LONGEST COMMON SUBSEQUENCE

INSTANCE: Finite alphabet Σ, finite set R of strings from Σ^*.

SOLUTION: A string $w \in \Sigma^*$ such that w is a subsequence of each $x \in R$, i.e. one can get w by taking away letters from each x.

MEASURE: Length of the subsequence, i.e., $|w|$.

Bad News: Not approximable within $|\Sigma|^{1/6-\varepsilon}$ for any $\varepsilon > 0$ [Bellare and Sudan, 1994].
Comment: Transformation from MAXIMUM INDEPENDENT SET with an alphabet Σ of the same size as the set of vertices in the MAXIMUM INDEPENDENT SET problem [Berman and Schnitger, 1992]. Not approximable within $|\Sigma|^{1/4-\varepsilon}$ for any $\varepsilon > 0$, unless QNP⊆co-QR [Bellare and Sudan, 1994]. APX-complete if the size of the alphabet Σ is fixed [Jiang and Li, 1994] and [Bonizzoni, Duella, and Mauri, 1994].
Garey and Johnson: SR10

Sequencing and Scheduling

Sequencing on One Processor

SS1. MAXIMUM CONSTRAINED SEQUENCING TO MINIMIZE TARDY TASK WEIGHT

INSTANCE: Set T of tasks, for each task $t \in T$ a length $l(t) \in Z^+$, a weight $w(t) \in Z^+$, and a deadline $d(t) \in Z^+$, a subset $S \subseteq T$, and a positive integer K.
SOLUTION: A one-processor schedule σ for T such that the sum of $w(t)$, taken over all $t \in T$ for which $\sigma(t) + l(t) > d(t)$ does not exceed K.
MEASURE: Cardinality of jobs in S completed by the deadline.

Bad News: Not approximable within $|T|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: Similar to SS3

SS2. MINIMUM STORAGE-TIME SEQUENCING

INSTANCE: Set T of tasks, a directed acyclic graph $G = \langle T, E\rangle$ defining preceding constraints for the tasks, for each task a length $l(t) \in Z^+$, and for each edge in the graph a weight $w(t_1, t_2)$ measuring the storage required to save the intermediate results generated by task t_1 until it is consumed by task t_2.
SOLUTION: A one-processor schedule for T that obeys the preceding constraints, i.e., a permutation $\pi : [1..|T|] \to [1..|T|]$ such that, for each edge $\langle t_i, t_j\rangle \in E$, $\pi^{-1}(i) < \pi^{-1}(j)$.
MEASURE: The total storage-time product, i.e.,

$$\sum_{\langle t_{\pi(i)}, t_{\pi(j)}\rangle \in E} w(t_{\pi(i)}, t_{\pi(j)}) \sum_{k=\min(i,j)}^{\max(i,j)} l(t_{\pi(k)}).$$

Good News: Approximable within $O\left(\log |T| \log \sum_{t\in T} l(t)\right)$ [Ravi, Agrawal, and Klein, 1991].

Multiprocessor Scheduling

SS3. MINIMUM MULTIPROCESSOR SCHEDULING

INSTANCE: Set T of tasks, number m of processors, length $l(t, i) \in Z^+$ for each task $t \in T$ and processor $i \in [1..m]$.
SOLUTION: An m-processor schedule for T, i.e., a function $f : T \to [1..m]$.

Measure: The finish time for the schedule, i.e., $\max_{i\in[1..m]} \sum_{\substack{t\in T:\\ f(t)=i}} l(t,i)$.

Good News: Approximable within 2 [Lenstra, Shmoys, and Tardos, 1990].

Bad News: Not approximable within within $3/2-\varepsilon$ for any $\varepsilon > 0$ [Lenstra, Shmoys, and Tardos, 1990].

Comment: Admits an FPTAS for the variation in which the number of processors m is constant [Horowitz and Sahni, 1976]. Admits a PTAS for the uniform variation, in which $l(t,i)$ is independent of the processor i [Hochbaum and Shmoys, 1987]. A variation in which, for each task t and processor i, a cost $c(t,i)$ is given in input and the goal is to minimize a weighted sum of the finish time and the cost is approximable within 2 [Shmoys and Tardos, 1993].

Garey and Johnson: SS8

SS4. Minimum Precedence Constrained Scheduling

Instance: Set T of tasks, each having length $l(t) = 1$, number m of processors, and a partial order $<$ on T.

Solution: An m-processor schedule for T that obeys the precedence constraints, i.e., a function $f : T \to N$ such that, for all $u \le 0$, $|f^{-1}(u)| \le m$ and such that $t < t'$ implies $f(t') > f(t)$.

Measure: The finish time for the schedule, i.e., $\max_{t\in T} f(t)$.

Good News: Approximable within $2 - 2/|T|$ [Lam and Sethi, 1977].

Comment: A variation with an enlarged class of allowable constraints is approximable within $3 - 4/(|T| + 1)$ while a variation in which the partial order $<$ is substituted with a weak partial order $\le$ is approximable within $2 - 2/(|T| + 1)$ [Berger and Cowen, 1991].

Garey and Johnson: SS9

SS5. Minimum Resource Constrained Scheduling

Instance: Set T of tasks each having length $l(t)$, number m of processors, number r of resources, resource bounds b_i, $1 \le i \le r$, and resource requirement $r_i(t)$, $0 \le r_i(t) \le b_i$, for each task t and resource i.

Solution: An m processor schedule for T that obeys the resource constraints, i.e., a function $f : T \to Z$ such that for all $u \le 0$, if $S(u)$ is the set of tasks t for which $f(t) \le u < f(t)+l(t)$, then $|S(u)| \le m$ and for each resource i

$$\sum_{t\in S(u)} r_i(t) \le b_i.$$

Measure: The makespan of the schedule, i.e., $\max_{t\in T}(f(t) + l(t))$.

Good News: Approximable within 2 [Garey and Graham, 1975].

Comment: Note that the restriction in which there is only one resource, i.e., the available processors, is identical to minimizing the makespan of the schedule of parallel tasks on m processors. In this case, minimizing the average response time, i.e., $\frac{1}{|T|}\sum_{t\in T}(f(t) + l(t))$ is approximable within 32 [Turek, Schwiegelshohn, Wolf, and Yu, 1994]. The further variation in which each task can be executed by any number of processors and the length of a task is a function of the number of processors allotted to it is also approximable [Ludwig and Tiwari, 1994].

Garey and Johnson: SS10

SS6. Minimum Preemptive Scheduling with Set-Up Times

Instance: Set T of tasks, number m of processors, length $l(t) \in Z^+$ and set-up time $s(t) \in Z^+$ for each $t \in T$.

Solution: An m-processor preemptive schedule for T, i.e., a partition of each task t into any number of subtasks $t_1, \ldots, t_k$ such that $\sum_{i=1}^{k} l(t_i) = l(t)$ and a schedule σ that, for all $t \in T$, assigns to each subtask t_i of t a positive integer $\sigma(t_i)$ such that $\sigma(t_{i+1}) \geq \sigma(t_i) + l(t_i) + s(t)$ and, for all $u > 0$, the number of subtasks for which $\sigma(t_i) \leq u < \sigma(t_i) + l(t_i) + s(t)$ is no more than m.

Measure: The overall completion time, i.e., the maximum over all subtasks of $\sigma(t_i) + l(t_i) + s(t)$.

Good News: Approximable within $3/2 - 1/(4m-4)$ for $m \leq 4$ and within $5/3 - 1/m$ for $m = 3j$, $j \geq 2$ [Monma and Potts, 1993].

Comment: If all set-up times are equal, then the problem is approximable within $3/2 - 1/2m$ for $m \geq 2$ and admits an FPTAS for $m = 2$ [Wöginger and Yu, 1992].

Garey and Johnson: SS12

SS7. Minimum Multiprocessor Scheduling with Speed Factors

Instance: Set T of tasks, number m of processors, for each task $t \in T$ a length $l(t) \in Z^+$, and for each processor $i \in [1..m]$ a speed factor $s(i) \in Q$ such that $s(1) = 1$ and $s(i) \geq 1$ for every i.

Solution: An m-processor schedule for T, i.e., a function $f : T \to [1..m]$.

Measure: The finish time for the schedule, i.e., $\max_{i \in [1..m]} \sum_{\substack{t \in T: \\ f(t)=i}} l(t)/s(i)$.

Good News: Admits a PTAS [Hochbaum and Shmoys, 1988].

Bad News: Does not admit an FPTAS [Hochbaum and Shmoys, 1988].

Comment: Admits an FPTAS for the variation in which the number of processors m is constant [Horowitz and Sahni, 1976].

Garey and Johnson: SS13

SS8. Minimum 3-Dedicated Processor Scheduling

Instance: Set T of tasks, set P of 3 processors, and, for each task $t \in T$, a length $l(t) \in Z^+$ and a required subset of processors $r(t) \subseteq P$.

Solution: A schedule for T, i.e., a starting time function $s : T \to Z^+$ such that, for any two tasks t_1 and t_2 with $r(t_1) \cap r(t_2) \neq \emptyset$, either $s(t_1) + l(t_1) < s(t_2)$ or $s(t_2) + l(t_2) < s(t_1)$.

Measure: The makespan of the schedule, i.e., $max_{t \in T}(s(t) + r(t))$.

Good News: Approximable within 5/4 [Dell'Olmo, Speranza, and Tuza, 1993].

SS9. MINIMUM JOB SHOP SCHEDULING

INSTANCE: Number m of processors, set J of jobs, each $j \in J$ consisting of a sequence of n_j operations $o_{i,j}$ with $1 \leq i \leq n_j$, for each such operation a processor $p_{i,j}$ and a length $l_{i,j}$.

SOLUTION: A job shop schedule for J, i.e., a collection of one-processor schedules $f_p : \{o_{i,j} : p_{i,j} = p\} \to N$ such that $f_p(o_{i,j}) > f_p(o_{i',j'})$ implies $f_p(o_{i,j}) \geq f_p(o_{i',j'}) + l_{i',j'}$ and such that $f_p(o_{i+1,j}) \geq f_p(o_{i,j}) + l_{i,j}$.

MEASURE: The completion time of the schedule, i.e., $\max_{j \in J} f_p(o_{n_j,j}) + l_{n_j,j}$.

Good News: Approximable within $O(\log^2(mN))$ where $N = \max_{j\in J} n_j$ [Shmoys, Stein, and Wein, 1991].

Bad News: NP-complete in the strong sense. Hence, does not admit an FPTAS [Garey, Johnson, and Sethi, 1976].

Comment: Transformation from 3-PARTITION. If each job must be processed on each machine at most once, then the factor N can be deleted. The same results hold for the variation in which the operations must be processed in an order consistent to a particular partial order and for the variation in which there are different types of machines, for each type, there are a specified number of identical processors, and each operation may be processed on any processor of the appropriate type.

Garey and Johnson: SS18

Miscellaneous

SS10. MINIMUM FILE TRANSFER SCHEDULING

INSTANCE: A file transfer graph, i.e., a graph $G = \langle V, E\rangle$, a port constraint function $p : V \to N$ and a file length function $L : E \to N$.

SOLUTION: A file transfer schedule, i.e., a function $s : E \to N$ such that, for each vertex v and for each $t \in N$,

$$|\{u : (u, v) \in E \wedge s(e) \leq t \leq s(e) + L(e)\}| \leq p(v).$$

MEASURE: The makespan of the schedule, i.e., $\max_{e\in E}(s(e) + L(e))$.

Good News: Approximable within 2.5 [Coffman, Garey, Johnson, and Lapaugh, 1985].

Comment: Several special cases with better guarantees are also obtainable [Coffman, Garey, Johnson, and Lapaugh, 1985].

SS11. MINIMUM SCHEDULE LENGTH

INSTANCE: A network $N = \langle V, E, b, c\rangle$ where $G = \langle V, E\rangle$ is a graph, $b : V \to N$ is the vertex-capacity function, and $c : E \to N$ is the edge-capacity function, and a set T of tokens $t = \langle u, v, p\rangle$ where $u, v \in V$ and p is either a path from u to v or the empty set.

SOLUTION: A schedule S, i.e., a sequence $f_0, \ldots, f_l$ of configuration functions $f_i : T \to V$ such that

1. For any token $t = \langle u, v, p\rangle$, $f_0(t) = u$ and $f_l(t) = v$.
2. For any $0 \leq i \leq l - 1$ and for any token t, if $f_i(t) = v$ and $f_{i+1}(t) = w$ then (a) $(u, v) \in E$, (b) $|\{t' : f_i(t') = w\}| < b(w)$, (c) $|\{t' : f_{i+1}(t') = w\}| \leq b(w)$, and (d) $|\{t' : f_i(t') = v \wedge f_{i+1}(t') = w\}| \leq c(w)$.

MEASURE: The length of the schedule, i.e., l.

Bad News: Not in APX [Clementi, and Di Ianni, 1994].
Comment: It remains non-approximable even for layered graphs.

SS12. MINIMUM VEHICLE SCHEDULING ON TREE

INSTANCE: Rooted tree $T = \langle V, E, v_0 \rangle$, a forward travel time $f : E \to N$, a backward travel time $b : E \to N$, a release time $r : V \to N$, and an handling time $h : V \to N$.
SOLUTION: A vehicle routing schedule that starts from v_0, visits all nodes of T, returns to v_0, and, for any node v_i, starts processing v_i not before the release time $r(v_i)$, i.e., a permutation π of $1, \ldots, |V|$ and a waiting function w such that, for any i,

$$d(v_0, v_{\pi(1)}) + \sum_{j=1}^{i-1} [w(v_{\pi(j)}) + h(v_{\pi(j)}) + d(v_{\pi(j)}, v_{\pi(j+1)})] \geq r(v_{\pi(i)})$$

where $d(u, v)$ denotes the length of the unique path from u to v.
MEASURE: The total completion time, i.e.,

$$d(v_0, v_{\pi(1)}) + \sum_{j=1}^{n-1} [w(v_{\pi(j)}) + h(v_{\pi(j)}) + d(v_{\pi(j)}, v_{\pi(j+1)})] + w(v_{\pi(n)}) + h(v_{\pi(n)}) + d(v_{\pi(n)}, v_0).$$

Good News: Approximable within 2 [Karuno, Nagamochi, and Ibaraki, 1993].

Mathematical Programming

MP1. MINIMUM $0-1$ PROGRAMMING

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$, nonnegative integer n-vector $c \in N^n$.
SOLUTION: A binary n-vector $x \in \{0,1\}^n$ such that $Ax \geq b$.
MEASURE: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Bad News: NPO-complete [Orponen and Mannila, 1987].
Comment: Transformation from MINIMUM WEIGHTED SATISFIABILITY. Variation in which $c_i = 1$ for all i is NPO PB-complete and not approximable within $n^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1993]. Variation in which there are at most two non-zero entries on each row of the matrix is approximable within 2 [Hochbaum, Megiddo, Naor, and Tamir, 1993].
Garey and Johnson: MP1

MP2. MAXIMUM BOUNDED $0-1$ PROGRAMMING

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$, nonnegative binary n-vector $c \in \{0,1\}^n$.
SOLUTION: A binary n-vector $x \in \{0,1\}^n$ such that $Ax \leq b$.

MEASURE: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Bad News: NPO PB-complete [Berman and Schnitger, 1992].

Comment: Transformation from LONGEST PATH WITH FORBIDDEN PAIRS. Not approximable within $n^{0.5-\varepsilon}$ for any $\varepsilon > 0$ [Crescenzi, Kann, and Trevisan, 1994].

Garey and Johnson: MP1

MP3. MAXIMUM QUADRATIC PROGRAMMING

INSTANCE: Positive integer n, set of linear constraints, given as an $m \times n$-matrix A and an m-vector b, specifying a region $S \subseteq R^n$ by $S = \{x \in [0,1]^n : Ax \le b\}$.

SOLUTION: A multivariate polynomial $f(x_1, \ldots, x_n)$ of total degree at most 2.

MEASURE: The maximum value of f in the region specified by the linear constants, i.e., $\max_{x \in S} f(x)$.

Bad News: Does not admit a μ-approximation for any constant $0 < \mu < 1$ [Bellare and Rogaway, 1993].

Comment: A μ-approximation algorithm finds a solution that differs from the optimal solution by at most the value $\mu\left(\max_{x \in S} f(x) - \min_{x \in S} f(x)\right)$. Variation in which we look for a polynomial f of any degree does not admit a μ-approximation for $\mu = 1 - n^{-\delta}$ for some $\delta > 0$ [Bellare and Rogaway, 1993]. Note that these problems are known to be solvable in polynomial space but are not known to be in NP.

Garey and Johnson: MP2

MP4. MINIMUM GENERALIZED $0-1$ ASSIGNMENT

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$, and binary $m \times n$-matrix $C \in \{0,1\}^{m \cdot n}$.

SOLUTION: A binary $m \times n$-matrix $X \in \{0,1\}^{m \cdot n}$ such that there is exactly one 1 in each column of X, and $\sum_{j=1}^{n} A_{i,j} X_{i,j} \le b_i$ for all $i \in [1..m]$.

MEASURE: $\sum_{i=1}^{m} \sum_{j=1}^{n} C_{i,j} X_{i,j}$.

Bad News: Not in APX [Sahni and Gonzalez, 1976].

MP5. MINIMUM QUADRATIC $0-1$ ASSIGNMENT

INSTANCE: Nonnegative integer $n \times n$-matrix $C \in N^{n \cdot n}$, nonnegative integer $m \times m$-matrix $D \in N^{m \cdot m}$.

SOLUTION: Binary $n \times m$-matrix $X \in \{0,1\}^{n \cdot m}$ such that there is at most one 1 in each row of X and exactly one 1 in each column of X.

MEASURE: $\sum_{\substack{i,j=1 \\ i \ne j}}^{n} \sum_{\substack{k,l=1 \\ k \ne l}}^{m} C_{i,j} D_{k,l} X_{i,k} X_{j,l}$.

Bad News: Not in APX [Sahni and Gonzalez, 1976].

Comment: Not in APX even if D satisfies the triangle inequality [Queyranne, 1986].

MP6. MINIMUM PLANAR RECORD PACKING

INSTANCE: Collection C of n records, for each record $c \in C$ a probability $p(c)$ such that $0 \le p(c) \le 1$.

SOLUTION: For each record $c \in C$ a placement $z(c)$ in the plane, given as integer coordinates, such that all records are placed on different points in the plane.

MEASURE: $\sum_{c_1 \in C} \sum_{c_2 \in C} p(c_1)p(c_2)d(z(c_1), z(c_2))$, where $d(z(c_1), z(c_2))$ is the discretized Euclidean distance between the points $z(c_1)$ and $z(c_2)$.

Good News: Approximable with an absolute error guarantee of $\lfloor 4\sqrt{2} + 8\sqrt{\pi} \rfloor$, that is, one can in polynomial time find a solution with objective function value at most $opt + \lfloor 4\sqrt{2} + 8\sqrt{\pi} \rfloor$ [Karp, McKellar, and Wong, 1975].

MP7. MINIMUM RELEVANT VARIABLES IN LINEAR SYSTEM

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$.

SOLUTION: A rational n-vector $x \in Q^n$ such that $Ax = b$.

MEASURE: The number of non-zero elements in x.

Bad News: Not in APX [Amaldi and Kann, 1994b].

Comment: Not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP⊂QP [Amaldi and Kann, 1994b]. The above nonapproximability results are still true for the variation in which the solutions are restricted by $Ax \ge b$ instead of $Ax = b$. Variation in which the solution vector is restricted to contain binary numbers is NPO PB-complete and is not approximable within $n^{0.5-\varepsilon}$ for any $\varepsilon > 0$ [Amaldi and Kann, 1994b]. The corresponding maximization problem, where the number of zero elements in the solution is to be maximized, and the solution vector is restricted to contain binary numbers, is NPO PB-complete and is not approximable within $n^{1/3-\varepsilon}$ for any $\varepsilon > 0$ [Crescenzi, Kann, and Trevisan, 1994].

Garey and Johnson: MP5

MP8. MAXIMUM SATISFYING LINEAR SUBSYSTEM

INSTANCE: System $Ax = b$ of linear equations, where A is an integer $m \times n$-matrix, and b is an integer m-vector.

SOLUTION: A rational n-vector $x \in Q^n$.

MEASURE: The number of equations that are satisfied by x.

Bad News: Not approximable within m^ε for some $\varepsilon > 0$ [Amaldi and Kann, 1994a].

Comment: For any prime q the problem over GF[q] is approximable within q, but is not approximable within q^ε for some $\varepsilon > 0$. If the system consists of relations ($>$ or $\ge$) the problem is APX-complete and approximable within 2 [Amaldi and Kann, 1994a]. If the variables are restricted to assume only binary values, the problem is harder to approximate than MAXIMUM INDEPENDENT SET. Approximability results for more variants of the problem can be found in [Amaldi and Kann, 1993].

MP9. MINIMUM UNSATISFYING LINEAR SUBSYSTEM

INSTANCE: System $Ax = b$ of linear equations, where A is an integer $m \times n$-matrix, and b is an integer m-vector.

SOLUTION: A rational n-vector $x \in Q^n$.
MEASURE: The number of equations that are *not* satisfied by x.

Bad News: Not in APX [Arora, Babai, Stern, and Sweedyk, 1993].
Comment: Not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP⊂QP [Arora, Babai, Stern, and Sweedyk, 1994]. If the system consists of relations ($>$ or $\geq$) the problem is even harder to approximate; there is a transformation from MINIMUM DOMINATING SET to this problem. If the variables are restricted to assume only binary values the problem is NPO PB-complete both for equations and relations, and is not approximable within $n^{1-\varepsilon}$ for any $\varepsilon > 0$. Approximability results for even more variants of the problem can be found in [Amaldi and Kann, 1994b].

MP10. MAXIMUM HYPERPLANE CONSISTENCY

INSTANCE: Finite sets P and N of integer n-vectors. P consists of positive examples and N of negative examples.
SOLUTION: A hyperplane specified by a normal vector $w \in Q^n$ and a bias w_0.
MEASURE: The number of examples that are consistent with respect to the hyperplane, i.e., $|\{x \in P : wx > w_0\}| + |\{x \in N : wx < w_0\}|$.

Good News: Approximable within 2 [Amaldi and Kann, 1994a].
Bad News: APX-complete [Amaldi and Kann, 1994a].
Comment: Variation in which only one type of misclassification, either positive or negative, is allowed is not approximable within n^ε for some $\varepsilon > 0$ [Amaldi and Kann, 1993]. The corresponding minimization problem, where the number of misclassifications is to be minimized, is not in APX unless P = NP, and is not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP⊂QP [Arora, Babai, Stern, and Sweedyk, 1994] and [Amaldi and Kann, 1994b].
Garey and Johnson: Similar to MP6

MP11. MAXIMUM KNAPSACK

INSTANCE: Finite set U, for each $u \in U$ a size $s(u) \in Z^+$ and a value $v(u) \in Z^+$, a positive integer $B \in Z^+$.
SOLUTION: A subset $U' \subseteq U$ such that $\sum_{u \in U'} s(u) \leq B$.
MEASURE: Total weight of the chosen elements, i.e., $\sum_{u \in U'} v(u)$.

Good News: Admits an FPTAS [Ibarra and Kim, 1975].
Garey and Johnson: MP9

MP12. MAXIMUM INTEGER m-DIMENSIONAL KNAPSACK

INSTANCE: Nonnegative integer $m \times n$-matrix, $A \in N^{m \cdot n}$, nonnegative integer m-vector $b \in N^m$, nonnegative integer n-vector $c \in N^n$.
SOLUTION: Nonnegative integer n-vector $x \in N^n$ such that $Ax \leq b$.
MEASURE: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Good News: Admits a PTAS [Chandra, Hirschberg, and Wong, 1976].
Garey and Johnson: Similar to MP10

MP13. Maximum Integer k-Choice Knapsack

Instance: Nonnegative integer $n \times k$-matrices $A, C \in N^{n \cdot k}$, nonnegative integer $b \in N$.
Solution: Nonnegative integer vector $x \in N^n$, function $f : [1..n] \to [1..k]$ such that $\sum_{i=1}^{n} a_{i,f(i)} x_i \leq b$.
Measure: $\sum_{i=1}^{n} c_{i,f(i)} x_i$.

Good News: Admits an FPTAS [Chandra, Hirschberg, and Wong, 1976].
Garey and Johnson: Similar to MP11

MP14. Nearest Lattice Vector

Instance: Lattice basis $\{b_1, \ldots, b_m\}$ where $b_i \in Z^k$, a point $b_0 \in Q^k$, and a positive integer p.
Solution: A vector b in the lattice, where $b \neq b_0$.
Measure: The distance between b_0 and b in the ℓ_p norm.

Bad News: Not in Apx [Arora, Babai, Stern, and Sweedyk, 1993].
Comment: Not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP$\subset$QP [Arora, Babai, Stern, and Sweedyk, 1994]. The special case where b_0 is the zero vector and $p = \infty$ is not approximable within $2^{\log^{0.5-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP$\subset$QP [Arora, Babai, Stern, and Sweedyk, 1993].

MP15. Minimum Block-angular Convex Programming

Instance: K disjoint convex compact sets B^k called blocks, M nonnegative continuous convex functions $f_m^k : B^k \to R$.
Solution: A positive number λ such that

$$\sum_{k=1}^{K} f_m^k(x^k) \leq \lambda \text{ for } 1 \leq m \leq M, \text{ and } x^k \in B^k \text{ for } 1 \leq k \leq K.$$

Measure: λ

Good News: Admits an FPTAS [Grigoriadis and Khachiyan, 1994].

Algebra and Number Theory

Solvability of Equations

AN1. Maximum Satisfiability of Quadratic Equations over GF$[q]$

Instance: Prime number q, set $P = \{p_1(x), p_2(x), \ldots, p_m(x)\}$ of polynomials of degree at most 2 over GF$[q]$ in n variables. The polynomials may not contain any monomial ${x_i}^2$ for any i.

Solution: A subset $P' \subseteq P$ of the polynomials such that there is a root common to all polynomials in P'.

Measure: Cardinality of the subset, i.e., $|P'|$.

Good News: Approximable within $q^2/(q-1)$ [Håstad, Phillips, and Safra, 1993].

Bad News: Not approximable within $q - \varepsilon$ for any ε [Håstad, Phillips, and Safra, 1993].

Comment: Over the rationals or over the reals the problem is not approximable within $n^{1-\varepsilon}$ for any $\varepsilon > 0$ [Håstad, Phillips, and Safra, 1993]. For linear polynomials the problem is not approximable within q^ε for some $\varepsilon > 0$ [Amaldi and Kann, 1994a].

Logic

Propositional Logic

LO1. Maximum Satisfiability

Instance: Set U of variables, collection C of disjunctive clauses of literals, where a literal is a variable or a negated variable in U.

Solution: A subset $C' \subseteq C$ of the clauses such that there is a truth assignment for U that satisfies every clause in C'.

Measure: Number of satisfied clauses, i.e., $|C'|$.

Good News: Approximable within 1.325 [Goemans and Williamson, 1994].

Bad News: Apx-complete [Papadimitriou and Yannakakis, 1991].

Comment: Variation in which each clause has a nonnegative weight and the objective is to maximize the total weight of the satisfied clauses is approximable within 4/3 [Yannakakis, 1992]. Generalization in which each clause is a disjunction of conjunctions of literals and each conjunction consists of at most k literals, where k is a positive constant, is still Apx-complete [Papadimitriou and Yannakakis, 1991].

Garey and Johnson: LO1

LO2. Maximum k-Satisfiability

Instance: Set U of variables, collection C of disjunctive clauses of at most k literals, where a literal is a variable or a negated variable in U. k is a constant, $k \geq 2$.

Solution: A subset $C' \subseteq C$ of the clauses such that there is a truth assignment for U that satisfies every clause in C'.

Measure: Number of satisfied clauses, i.e., $|C'|$.

Good News: Approximable within $1/(1 - 2^{-k})$ if every clause consists of exactly k literals [Johnson, 1974].

Bad News: Apx-complete [Papadimitriou and Yannakakis, 1991].

Comment: Maximum 3-Satisfiability is not approximable within 113/112 [Bellare, Goldwasser, Lund, and Russell, 1993]. Maximum 2-Satisfiability is approximable within 1.14 [Goemans and Williamson, 1994]. Admits a PTAS for 'planar' instances [Nishizeki and Chiba, 1988]. Variation in which the number of occurrences of any literal is bounded by the constant B is still Apx-complete [Papadimitriou and Yannakakis, 1991].
Garey and Johnson: LO2 and LO5

LO3. Minimum k-Satisfiability

Instance: Set U of variables, collection C of disjunctive clauses of at most k literals, where a literal is a variable or a negated variable in U. k is a constant, $k \geq 2$.
Solution: A subset $C' \subseteq C$ of the clauses such that there is a truth assignment for U that satisfies every clause in C'.
Measure: Number of satisfied clauses, i.e., $|C'|$.

Good News: Approximable within k [Kohli, Krishnamurti, and Mirchandani, 1994].
Garey and Johnson: LO2

LO4. Maximum Not-All-Equal 3-Satisfiability

Instance: Set U of variables, collection C of disjunctive clauses of 3 literals, where a literal is a variable or a negated variable in U.
Solution: A truth assignment for U and a subset $C' \subseteq C$ of the clauses such that each clause in C' has at least one true literal and at least one false literal.
Measure: $|C'|$

Good News: Approximable within a constant [Papadimitriou and Yannakakis, 1991].
Bad News: Apx-complete [Papadimitriou and Yannakakis, 1991].
Comment: Transformation from Maximum 2-Satisfiability.
Garey and Johnson: LO3

LO5. Minimum 3DNF Satisfiability

Instance: Set U of variables, collection C of conjunctive clauses of at most three literals, where a literal is a variable or a negated variable in U.
Solution: A subset $C' \subseteq C$ of the clauses such that there is a truth assignment for U that satisfies every clause in C'.
Measure: Number of satisfied clauses, i.e., $|C'|$.

Bad News: Not in Apx [Kolaitis and Thakur, 1993].
Garey and Johnson: LO8

LO6. Maximum Distinguished Ones

Instance: Disjoint sets X, Z of variables, collection C of disjunctive clauses of at most 3 literals, where a literal is a variable or a negated variable in $X \cup Z$.
Solution: Truth assignment for X and Z that satisfies every clause in C.

MEASURE: The number of Z variables that are set to true in the assignment.

Bad News: NPO PB-complete [Kann, 1992b].

Comment: Transformation from MAXIMUM NUMBER OF SATISFIABLE FORMULAS [Panconesi and Ranjan, 1993]. Not approximable within $|Z|^{0.5-\varepsilon}$ for any $\varepsilon > 0$ [Crescenzi, Kann, and Trevisan, 1994]. MAXIMUM ONES, the variation in which all variables are distinguished, i.e. $|X| = \emptyset$, is also NPO PB-complete [Kann, 1992b], and is not approximable within $|Z|^{1/3-\varepsilon}$ for any $\varepsilon > 0$ [Crescenzi, Kann, and Trevisan, 1994]. MAXIMUM WEIGHTED SATISFIABILITY, the weighted version, in which every variable is assigned a nonnegative weight, is NPO-complete.

LO7. MINIMUM DISTINGUISHED ONES

INSTANCE: Disjoint sets X, Z of variables, collection C of disjunctive clauses of at most 3 literals, where a literal is a variable or a negated variable in $X \cup Z$.

SOLUTION: Truth assignment for X and Z that satisfies every clause in C.

MEASURE: The number of Z variables that are set to true in the assignment.

Bad News: NPO PB-complete [Kann, 1993].

Comment: Transformation from MINIMUM INDEPENDENT DOMINATING SET. Not approximable within $|Z|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1993]. MINIMUM ONES, the variation in which all variables are distinguished, i.e. $|X| = \emptyset$, is also NPO PB-complete, and is not approximable within $|Z|^{0.5-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1993]. MINIMUM ONES for clauses of 2 literals is approximable within 2 [Gusfield and Pitt, 1992]. MINIMUM WEIGHTED SATISFIABILITY, the weighted version, in which every variable is assigned a nonnegative weight, is NPO-complete [Orponen and Mannila, 1987].

LO8. MAXIMUM WEIGHTED SATISFIABILITY WITH BOUND

INSTANCE: Set U of variables, boolean expression F over U, a nonnegative bound $B \in N$, for each variable $u \in U$ a weight $w(u) \in N$ such that $B \leq \sum_{u \in U} w(u) \leq 2B$.

SOLUTION: A truth assignment for U, i.e., a subset $U' \subseteq U$ such that the variables in U' are set to true and the variables in $U - U'$ are set to false.

MEASURE: $\sum_{v \in U'} w(v)$ if the truth assignment satisfies the boolean expression F and B otherwise.

Good News: Approximable within 2 [Crescenzi and Panconesi, 1991].

Bad News: APX-complete [Crescenzi and Panconesi, 1991].

Comment: Variation in which $\sum_{u \in U} w(u) \leq (1 + 1/(|U| - 1))\, B$ is PTAS-complete [Crescenzi and Panconesi, 1991].

LO9. MAXIMUM NUMBER OF SATISFIABLE FORMULAS

INSTANCE: Set U of variables, collection C of 3CNF formulas.

SOLUTION: A subset $C' \subseteq C$ of the formulas such that there is a truth assignment for U that satisfies every formula in C'.

MEASURE: Number of satisfied formulas, i.e., $|C'|$.

Bad News: NPO PB-complete [Kann, 1992b].
Comment: Transformation from LONGEST INDUCED PATH. Not approximable within $|C|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Crescenzi, Kann, and Trevisan, 1994].

LO10. MINIMUM NUMBER OF SATISFIABLE FORMULAS

INSTANCE: Set U of variables, collection C of 3CNF formulas.
SOLUTION: A subset $C' \subseteq C$ of the formulas such that there is a truth assignment for U that satisfies every formula in C'.
MEASURE: Number of satisfied formulas, i.e., $|C'|$.

Bad News: NPO PB-complete [Kann, 1993].
Comment: Transformation from MINIMUM DISTINGUISHED ONES. Not approximable within $|C|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1993].

LO11. MINIMUM EQUIVALENCE DELETION

INSTANCE: Set U of variables, collection C of equivalences, i.e., pairs of literals over U.
SOLUTION: A subset $C' \subseteq C$ of the formulas such that there is a truth assignment for U that satisfies every equivalence in C'.
MEASURE: Number of equivalences that are not satisfied, i.e., $|C| - |C'|$.

Good News: Approximable within $O(\log n)$ [Garg, Vazirani, and Yannakakis, 1993b].
Bad News: APX-hard [Garg, Vazirani, and Yannakakis, 1993b].

LO12. MAXIMUM k-CONSTRAINT SATISFACTION

INSTANCE: Set U of variables, collection C of conjunctive clauses of at most k literals, where a literal is a variable or a negated variable in U, and k is a constant, $k \geq 2$.
SOLUTION: A subset $C' \subseteq C$ of the clauses such that there is a truth assignment for U that satisfies every clause in C'.
MEASURE: Number of satisfied clauses, i.e., $|C'|$.

Good News: Approximable within 2^k [Berman and Schnitger, 1992].
Bad News: APX-complete [Berman and Schnitger, 1992].
Comment: Transformation from MAXIMUM 2-SATISFIABILITY. Not approximable within $2^{o(k)}$ when $k \leq \log |C|$ [Berman and Schnitger, 1992].

Miscellaneous

LO13. MAXIMUM HORN CORE

INSTANCE: Set M of truth assignments on n variables.
SOLUTION: A Horn core of M, i.e., a subset $M' \subseteq M$ such that M' is equal to the set of truth assignments satisfying a Horn boolean formula.
MEASURE: The cardinality of the core, i.e., $|M'|$.

Bad News: Not in APX [Kavvadias, Papadimitriou, and Sideri, 1993].

Automata and Language Theory

Automata Theory

AL1. MINIMUM CONSISTENT FINITE AUTOMATON

INSTANCE: Two finite sets of binary strings P, N.
SOLUTION: $S(\langle P, N\rangle) = \{A = \langle Q, \{0,1\}, \delta, q_0, F\rangle$ A deterministic finite automaton accepting all strings in P and rejecting all strings in N.
MEASURE: Number of states in the automaton.
Bad News: Not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Simon, 1990].
Comment: Transformation from MINIMUM GRAPH COLORING. Not approximable within $(|P| + |N|)^{1/14-\varepsilon}$ for any $\varepsilon > 0$ [Pitt and Warmuth, 1993].
Garey and Johnson: AL8

AL2. LONGEST COMPUTATION

INSTANCE: Nondeterministic Turing machine M, binary input string x.
SOLUTION: Nondeterministic guess string c produced by M on input x.
MEASURE: The length of the shortest of the strings c and x, i.e., $\min(|c|, |x|)$.

Bad News: NPO PB-complete [Berman and Schnitger, 1992].
Comment: Not approximable within n^{ε} for some $\varepsilon > 0$, where n is the size of the input [Berman and Schnitger, 1992]. Variation in which the Turing machine is oblivious is also NPO PB-complete.

AL3. SHORTEST COMPUTATION

INSTANCE: Nondeterministic Turing machine M, binary input string x.
SOLUTION: Nondeterministic guess string c produced by M on input x.
MEASURE: The length of the shortest of the strings c and x, i.e., $\min(|c|, |x|)$.

Bad News: NPO PB-complete [Kann, 1993].
Comment: Not approximable within n^{ε} for some $\varepsilon > 0$, where n is the size of the input [Kann, 1993]. Variation in which the Turing machine is oblivious is also NPO PB-complete.

Formal Languages

AL4. MINIMUM LOCALLY TESTABLE AUTOMATON ORDER

INSTANCE: A locally testable language L, i.e., a language L such that, for some positive integer j, whether or not a string x is in the language depends on (a) the prefix and suffix of x of length $j - 1$, and (b) the set of substrings of x of length j.
SOLUTION: An order j, i.e., a positive integer j witnessing the local testability of L.
MEASURE: The value of the order, i.e., j.
Good News: Admits a PTAS [Kim and McNaughton, 1993].

Miscellaneous

AL5. Minimum Permutation Group Base

Instance: Permutation group on n letters.
Solution: A base for G, i.e., a sequence of points $b_1, \ldots, b_k$ such that the only element in G fixing all of the b_i is the identity.
Measure: The size of the base, i.e., k.
Good News: Approximable within $\log\log n$ [Blaha, 1992].

Program Optimization

Code Generation

PO1. Minimum Register Sufficiency

Instance: Directed acyclic graph $G = \langle V, E\rangle$.
Solution: Computation for G that uses k register, i.e., an ordering $v_1, \ldots, v_n$ of the vertices in V and a sequence $S_0, \ldots, S_n$ of subsets of V, each satisfying $|S_i| \le k$, such that S_0 is empty, S_n contains all vertices with in-degree 0 in G, and, for $1 \le i \le n$, $v_i \in S_i$, $S_i - \{v_i\} \subseteq S_{i-1}$, and S_{i-1} contains all vertices u for which $(v_i, u) \in A$.
Measure: Number of registers, i.e., k.
Good News: Approximable within $O(\log^2 n)$ [Klein, Agrawal, Ravi, and Rao, 1990]].
Garey and Johnson: PO1

Miscellaneous

MS1. Nearest Codeword

Instance: Linear binary code C of length n and a string x of length n.
Solution: A codeword y of C.
Measure: The Hamming distance between x and y, i.e., $d(x, y)$.
Bad News: Not in Apx [Arora, Babai, Stern, and Sweedyk, 1993].
Comment: Not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP⊂QP [Arora, Babai, Stern, and Sweedyk, 1994]. The corresponding maximization problem, where the number of bits that agree between x and y is to be maximized, does not admit a PTAS [Petrank, 1993].

MS2. Attraction Radius for Binary Hopfield Net

Instance: An n-node synchronous binary Hopfield network and a stable initial vector of states $u \in \{-1, 1\}^n$. A binary Hopfield network is a complete graph where each edge has an integer weight $w(v_i, v_j)$ and each vertex has an integer threshold value $t(v_i)$. At each time step t each vertex v_i has a state $x(t, v_i)$. $x(0, v_i)$ is given by u and $x(t+1, v_i) = \text{sgn}\left(\sum_{j=1}^{n} w(v_i, v_j) - t_i\right)$ where sgn is the sign function. An initial vector of states is stable if $x(t, v_i)$ eventually converges for all i.
Solution: An initial vector of states v that either converges to a different vector than u or is not stable.

MEASURE: The Hamming distance between u and v. If v is the vector nearest to u that does not converge to the same vector as u, then this distance is the attraction radius.
Bad News: Not approximable within $n^{1-\varepsilon}$ for any ε [Floréen and Orponen, 1993].
Comment: Transformation from MINIMUM INDEPENDENT DOMINATING SET.

MS3. MINIMUM k-CLUSTERING

INSTANCE: Finite set X, a distance $d(x, y) \in N$ for each pair $x, y \in X$. The distances must satisfy the triangle inequality.
SOLUTION: A partition of X into disjoint subsets $C_1, C_2, \ldots, C_k$.
MEASURE: The largest distance between two elements in the same subset, i.e., $\max_{\substack{i\in[1..k]\\ x,y\in C_i}} d(x, y)$.

Good News: Approximable within 2 [Hochbaum and Shmoys, 1986].
Bad News: Not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].
Garey and Johnson: MS9

MS4. MINIMUM k-CLUSTERING SUM

INSTANCE: Finite set X, a distance $d(x, y) \in N$ for each pair $x, y \in X$.
SOLUTION: A partition of X into disjoint subsets $C_1, C_2, \ldots, C_k$.
MEASURE: The sum of all distances between elements in the same subset, i.e.,

$$\sum_{i=1}^{k} \sum_{v_1,v_2\in C_i} d(v_1, v_2).$$

Bad News: Not in APX [Sahni and Gonzalez, 1976].

MS5. MAXIMUM CHANNEL ASSIGNMENT

INSTANCE: Net of hexagonal cells in which n cells C_i are assigned a positive load r_i, an interference radius r, and m channels F_i.
SOLUTION: A channel assignment A, i.e., a multivalued function A assigning a set of cells to a channel such that if $C_i, C_j \in A(F_k)$ then the distance between C_i and C_j is greater than $2r$.
MEASURE: The number of satisfied request, i.e., $\sum_i \min\{r_i, |\{F_j : C_i \in F_j\}|\}$.
Good News: Approximable within $1/(1 - e^{-1})$ [Simon, 1989].
Comment: Admits a PTAS if the number of channels is fixed. Similar results hold in the case in which each cell has a set of forbidden channels.

MS6. MINIMUM k-LINK PATH IN A POLYGON

INSTANCE: Polygon P with n integer-coordinates vertices and two points s and t in P.
SOLUTION: A k-link path between s and t, i.e., a sequence $p_0, \ldots, p_h$ of points inside P with $h \le k$ such that $p_0 = s$, $p_k = t$, and, for all i with $0 \le i < h$, the segment between p_i and p_{i+1} is inside P.
MEASURE: The Euclidean length of the path, i.e., $\sum_{i=0}^{h-1} d(p_i, p_{i+1})$ where d denotes the Euclidean distance.
Good News: Admits an FPTAS [Mitchell, Piatko, and Arkin, 1992].

MS7. Minimum Size Ultrametric Tree

Instance: $n \times n$ matrix M of positive integers.

Solution: An ultrametric tree, i.e., an edge-weighted tree $T(V,E)$ with n leaves such that, for any pair of leaves i and j, $d^T_{ij} \geq M[i,j]$ where d^T_{ij} denotes the sum of the weights in the path between i and j.

Measure: The size of the tree, i.e., $\sum_{e \in E} w(e)$ where $w(e)$ denotes the weight of edge e.

Bad News: Not approximable within n^{ε} for a given $\varepsilon > 0$ [Farach, Kannan, and Warnow, 1993].

Comment: Transformation from graph coloring.

MS8. Minimum Partition of Rectangle with Interior Points

Instance: Rectangle R and finite set P of points located inside R.

Solution: A set of line segments that partition R into rectangles such that every point in P is on the boundary of some rectangle.

Measure: The total length of the introduced line segments.

Good News: Approximable within 3 [Gonzalez and Zheng, 1990].

Comment: Variation in which R is a rectilinear polygon is approximable within 4 [Gonzalez and Zheng, 1990].

MS9. Minimum Sorting by Reversals

Instance: Permutation π of the numbers 1 to n.

Solution: A sequence $\rho_1, \rho_2, \ldots, \rho_t$ of reversals of intervals such that $\pi \cdot \rho_1 \cdot \rho_2 \cdots \rho_t$ is the identity permutation. A reversal of an interval $[i,j]$ is the permutation $(1, 2, \ldots, i-1, j, -1, \ldots, i+1, i, j+1, \ldots, n)$.

Measure: The number of reversals, i.e., t.

Good News: Approximable within 7/4 [Bafna and Pevzner, 1993].

Comment: The problem is not known to be NP-complete. Variation in which the numbers are signed and a reversal of an interval changes the signs of the numbers in the interval is approximable within 3/2 [Bafna and Pevzner, 1993].

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