Robust single machine scheduling problem with weighted number of late jobs criterion

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Abstract This paper deals with a single machine scheduling problem with the weighted number of late jobs criterion, where some job parameters, such as: processing times, due dates, and weights, may be uncertain. This uncertainty is modeled by specifying a scenario set containing all vectors of the job parameters, called scenarios, which may occur. The min-max criterion is adopted to compute a solution under uncertainty. In this paper some of the recent negative complexity and approximability results for the problem are extended and strengthened. Moreover, some positive approximation results for the problem in which the maximum criterion is replaced with the OWA operator are presented.

1 Preliminaries

In a single machine scheduling problem with the weighted number of late jobs criterion, we are given a set of n independent, nonpreemptive, ready for processing at time 0 jobs, $J = \{J_1, \dots, J_n\}$, to be processed on a single machine. For each job $j \in J$ a processing time p_j, a due date d_j and a weight w_j are specified. A schedule π is a permutation of the jobs representing an order in which the jobs are processed. We will use Π to denote the set of all schedules. Let $C_j(\pi)$ denote the completion time of job j in schedule π. Job $j \in J$ is *late* in π if $C_j(\pi) > d_j$; otherwise j is *on-time* in π. Set $U_j(\pi) = 1$ if job j is late in π and $U_j(\pi) = 0$ if j is on-time in π, $U_j(\pi)$ is called the *unit penalty* of job j in π. In the deterministic case, we wish to find a schedule $\pi \in \Pi$ which minimizes the value of the *cost function* $f(\pi) = \sum_{j \in J} w_j U_j(\pi)$. This

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problem is denoted by $1||\sum w_jU_j$ in Graham's notation (see, e.g., [7]). The problem $1||\sum w_jU_j$ is weakly NP-hard [8]. However, its special cases $1||\sum U_j$ (minimizing the number of late jobs) and $1|p_j = 1|\sum w_jU_j$ (minimizing the weighted number of late jobs with unit processing times) are polynomially solvable (see, e.g., [4]).

Suppose that all the job parameters may be ill-known. Every possible realization of the parameters, denoted by S, is called a *scenario*. We will use $p_j(S), d_j(S)$ and $w_j(S)$ to denote the processing time, due date and weight of job j under scenario S, respectively. Without loss of generality, we can assume that all these parameters are nonnegative integers. Let *scenario set* $\Gamma = \{S_1,\dots,S_K\}$ contain K explicitly listed scenarios. Now the job completion time, the unit penalty and the cost of schedule π depend on scenario $S \in \Gamma$, and we will denote them by $C_j(\pi,S)$, $U_j(\pi,S)$ and $f(\pi,S)$, respectively. In order to compute a solution we will use the min-max criterion, which is the most popular criterion in *robust optimization* (see, e.g. [11]). Namely, in the MIN-MAX $1||\sum w_jU_j$ problem, we seek a schedule that minimizes the largest cost over all scenarios, that is

$$\min_{\pi\in\Pi}\max_{S\in\Gamma} f(\pi,S), \tag{1}$$

where $f(\pi,S) = \sum_{j\in J} w_j(S)U_j(\pi,S)$. We will also discuss its special cases, MIN-MAX $1||\sum U_j$ and MIN-MAX $1|p_j = 1|\sum U_j$, in which $f(\pi,S) = \sum_{j\in J} U_j(\pi,S)$ is the number of late jobs in π under scenario S.

2 Single machine scheduling problem with the number of late jobs criterion under uncertainty

In this section, we extend and strengthen the negative results which have been recently obtained in [1, 2] for some special cases of the MIN-MAX $1||\sum w_jU_j$ problem, namely for MIN-MAX $1||\sum U_j$ and MIN-MAX $1|p_j = 1|\sum U_j$. It has been proved in [2] that MIN-MAX $1||\sum U_j$ with deterministic due dates and uncertain processing times is weakly NP-hard, if the number of processing time scenarios equals 2. We now show that if the number of processing time scenarios is a part of the input, then the problem is strongly NP-hard even if all jobs have a common deterministic due date.

Theorem 1. *When the number of processing time scenarios is a part of the input, then* MIN-MAX $1||\sum U_j$ *is strongly NP-hard. This assertion remains true even when all the jobs have a common deterministic due date.*

Proof. We show a polynomial time reduction from the following 3-SAT problem which is strongly NP-hard [6]. Given a set of boolean variables $x_1,\dots,x_n$ and a set of clauses $C_1,\dots,C_m$, where each clause contains exactly three distinct literals (variables or their negations). We ask if there is a truth assignment to the variables which satisfies all the clauses. Given an instance of 3-SAT, we create an instance of MIN-MAX $1||\sum U_j$ in the following way. For each variable x_i we create two jobs

J_{x_i} and $J_{\overline{x}_i}$, so J contains $2n$ jobs. The due dates of all these jobs are the same under each scenario and equal 2. We form processing time scenario set Γ as follows. For each clause $C_j = (l_1, l_2, l_3)$ we construct scenario under which the jobs $J_{\overline{l}_1}, J_{\overline{l}_2}, J_{\overline{l}_3}$ have processing time equal to 1 and all the remaining jobs have processing times equal to 0. Then, for each pair of jobs $J_{x_i}, J_{\overline{x}_i}$ we construct scenario S'_i under which the processing times of $J_{x_i}, J_{\overline{x}_i}$ are 2 and all the remaining jobs have processing times equal to 0. A sample reduction is shown in Table 1. We will show that the answer to 3-SAT is yes if and only if there is a schedule π such that $\max_{S\in\Gamma} f(\pi,S) \le n$.

Table 1 Processing time scenarios for the formula $(x_1 \vee \overline{x}_2 \vee \overline{x}_3) \wedge (\overline{x}_2 \vee \overline{x}_3 \vee x_4) \wedge (\overline{x}_1 \vee x_2 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_3 \vee \overline{x}_4)$. Schedule $\pi = (J_{x_1}, J_{\overline{x}_2}, J_{x_3}, J_{\overline{x}_4} | J_{\overline{x}_1}, J_{x_2}, J_{\overline{x}_3}, J_{x_4})$ corresponds to a satisfying truth assignment.

	S_1	S_2	S_3	S_4	S_5	S'_1	S'_2	S'_3	S'_4	d_i
J_{x_1}	0	0	**1**	0	0	**2**	0	0	0	**2**
$J_{\overline{x}_1}$	**1**	0	0	**1**	**1**	**2**	0	0	0	**2**
J_{x_2}	**1**	**1**	0	0	0	0	**2**	0	0	**2**
$J_{\overline{x}_2}$	0	0	**1**	**1**	0	0	**2**	0	0	**2**
J_{x_3}	**1**	**1**	0	0	0	0	0	**2**	0	**2**
$J_{\overline{x}_3}$	0	0	0	**1**	**1**	0	0	**2**	0	**2**
J_{x_4}	0	0	**1**	0	**1**	0	0	0	**2**	**2**
$J_{\overline{x}_4}$	0	**1**	0	0	0	0	0	0	**2**	**2**

Assume that the answer to 3-SAT is yes. Then there exists a truth assignment to the variables which satisfies all the clauses. Let us form schedule π by processing first the jobs corresponding to true literals in any order and processing then the remaining jobs in any order. From the construction of the scenario set it follows that the completion time of the nth job in π under each scenario is not greater than 2. In consequence, at most n jobs in π is late under each scenario and $\max_{S\in\Gamma} f(\pi,S) \le n$.

Assume now that there is a schedule π such that $f(\pi,S) \le n$ for each $S \in \Gamma$ which means that at most n jobs in π are late under each scenario. Observe first that J_{x_i} and $J_{\overline{x}_i}$ cannot appear among the first n jobs in π for any $i \in [n]$; otherwise more than n jobs would be late in π under S'_i. Hence the first n jobs in π correspond to a truth assignment to the variables $x_1, \dots, x_n$, i.e. when J_l is among the first n jobs, then the literal l is true. Since $f(\pi,S) \le n$, the completion time of the n-th job in π is not greater than 2. We conclude that at most two jobs among the first n job have processing time equal to 1 under S, so there are at most two false literals for each clause and the answer to 3-SAT is yes. $\Box$

We now discuss the MIN-MAX $1|p_j = 1|\sum U_j$ problem under due date uncertainty. It has been proved in [1], that when the number of due date scenarios is a part of the input and there are two distinct due date values, the problem is strongly NP-hard and it is not approximable within 2. We now extend this result, namely, we show that if the number of due date scenarios is a part of the input and there are two distinct due date values, the problem is not approximable within any constant factor.

Consider the following 0-1 SELECTING ITEMS problem. We are given a set of items $E = \{e_1, e_2, \dots, e_n\}$ and an integer $p \in [n]$. For each item e_j, $j \in [n]$, there is a cost $c_j \in \{0,1\}$. We seek a selection $X \subset E$ of exactly p items of the minimum total cost $f(X) = \sum_{e_j \in X} c_j$.

Proposition 1. *There is a cost preserving reduction from* 0-1 SELECTING ITEMS *problem to* $1|p_j = 1|\sum U_j$.

Proof. Let $(E, p, (c_j)_{j\in[n]})$ be an instance of 0-1 SELECTING ITEMS. The corresponding scheduling problem is constructed as follows. We create a set of jobs $J = E$, $|E| = n$, with unit processing times. If $c_j = 1$ then $d_j = n - p$, and if $c_j = 0$, then $d_j = n$. Suppose that there is a selection X of p items out of E with the cost of C. Hence X contains exactly C items, $C \leq p$, with the cost equal to 1. In the corresponding schedule, we first process $n - p$ jobs from $J \setminus X$ and then the jobs in X in any order. It is easily seen that there are exactly C late jobs in π, hence the cost of schedule π is C. Let π be a schedule in which there are C late jobs. Clearly $C \leq p$ since the first $n - p$ jobs in π must be on-time. Let us form solution X by choosing the items corresponding to the last p jobs in π. Among these jobs exactly C are late, hence the cost of X is C. □

We have arrived to the theorem that improves the lower bound for approximating MIN-MAX $1|p_j = 1|\sum U_j$ give in [1].

Theorem 2. *When the number of due date scenarios is a part of the input,* MIN-MAX $1|p_j = 1|\sum U_j$ *is not approximable within any constant factor unless P=NP. This assertion remains true even if there are two distinct values of the due dates in scenarios.*

Proof. Proposition 1 shows that there is a cost preserving reduction from 0-1 SELECTING ITEMS to $1|p_j = 1|\sum U_j$. Therefore, there exists a cost preserving reduction from MIN-MAX 0-1 SELECTING ITEMS with K, 0-1 cost scenarios to MIN-MAX $1|p_j = 1|\sum U_j$ with K due date scenarios. Since the former problem is not approximable within any constant factor [9], the same results holds for the latter one. □

3 Single machine scheduling problem with the weighted number of late jobs criterion under uncertainty

In this section we explore the MIN-MAX $1|p_j = 1|\sum w_j U_j$ problem under due date and weight uncertainty. We note first that if the jobs have a common deterministic due date, then $1|p_j = 1|\sum w_j U_j$ is equivalent to the SELECTING ITEMS problem discussed in [3, 1, 5], i.e. a generalization of 0-1 SELECTING ITEMS (see Section 2) in which the items have arbitrary nonnegative costs. It suffices to fix $E = J$, $c_j = w_j$, $j \in J$, and $p = n - d$, where d is a common due date. The same reduction allows us to transform any instance of SELECTING ITEMS to MIN-MAX $1|p_j = 1|\sum w_j U_j$ with a deterministic common due date. Hence, there exists a cost preserving reduction from MIN-MAX $1|p_j = 1|\sum w_j U_j$ with K weight scenarios to MIN-MAX SELECTING

ITEMS with K cost scenarios and vice versa. Consequently, the results obtained in [1, 3] immediately imply the following theorem:

Theorem 3. *When the jobs have a common deterministic due date, then* MIN-MAX $1|p_j = 1|\sum w_j U_j$ *is NP-hard for two weight scenarios. Furthermore, it becomes strongly NP-hard and hard to approximate within any constant factor when the number of weight scenarios is part of the input.*

In [5], an LP-based $O(\log K/\log\log K)$ approximation algorithm for MIN-MAX SELECTING ITEMS has been proposed. Applying this algorithm to MIN-MAX $1|p_j = 1|\sum w_j U_j$ leads to the following result:

Theorem 4. *If the number of weight scenarios is a part of the input, then* MIN-MAX $1|p_j = 1|\sum w_j U_j$ *with a common deterministic due date is approximable within* $O(\log K/\log\log K)$.

We now consider the general case, in which both due dates and weights may be uncertain. We make use of the fact that that $1|p_j = 1|\sum w_j U_j$ is a special case of the MINIMUM ASSIGNMENT problem. To see this, we can build an instance $(G = (V_1 \cup V_2, E), (c_{ij})_{(i,j)\in E})$ of MINIMUM ASSIGNMENT for given an instance of $1|p_j = 1|\sum w_j U_j$ in the following way. The nodes in V_1 correspond to job positions, $V_1 = [n]$, the nodes in V_2 correspond to jobs, $V_2 = J$, obviously $|V_1| = |V_2| = n$. Each node $j \in V_2$ is connected with every node $i \in V_1$. The arc costs c_{ij}, $(i, j) \in E$, are set as follows: $c_{ij} = w_j$ if $i > d_j$, and $c_{ij} = 0$ otherwise. There is one to one correspondence between the schedules and the assignments and the reduction is cost preserving. This fact still holds, when a scenario set Γ is introduced. In this case we fix $c_{ij}(S) = w_j(S)$ if $i > d_j(S)$ and $c_{ij}(S) = 0$ otherwise for each scenario $S \in \Gamma$. The reduction is then cost preserving under each scenario. In consequence, $1|p_j = 1|\sum w_j U_j$ with scenario set Γ belongs to the class of combinatorial optimization problems discussed in [10], which allows us to establish a positive result described in the next part of this section.

Let $v_1, \ldots, v_K$ be numbers such that $v_i \in [0,1]$, $i \in [K]$, and $v_1 + \cdots + v_K = 1$. Given schedule π, let σ be a permutation of $[K]$ such that $f(\pi, S_{\sigma(1)}) \geq f(\pi, S_{\sigma(2)}) \geq \cdots \geq f(\pi, S_{\sigma(K)})$. The *Ordered Weighted Averaging* aggregation operator (OWA), introduced in [12], is defined as follows: $\mathrm{OWA}(\pi) = \sum_{i\in[K]} v_i f(\pi, S_{\sigma(i)})$. We now consider the following MIN-OWA $1|p_j = 1|\sum w_j U_j$ problem:

$$\min_{\pi\in\Pi} \mathrm{OWA}(\pi). \tag{2}$$

The choice of particular numbers v_i, $i \in [K]$, leads to well known criteria in decision making under uncertainty, among others: the maximum, the average and the Hurwicz pessimism - optimism criteria. Suppose that $v_1 \geq v_2 \geq \cdots \geq v_K$, such numbers are used if the idea of the robust optimization is adopted. Notice that this case contains both the maximum and the average criteria as special (boundary) cases. Indeed, if $v_1 = 1$ and $v_i = 0$ for $i \neq 1$, then we obtain the maximum criterion, the first extreme, and problem (2) becomes (1). If $v_i = 1/K$ for $i \in [K]$, then we get the average criterion - the second extreme. The following theorem holds:

Theorem 5. *If $v_1 \geq v_2 \geq \cdots \geq v_K$ then* MIN-OWA $1|p_j = 1|\sum w_j U_j$ *is approximable within $v_1 K$.*

Proof. The result follows from the fact that the problem is a special case of MIN-OWA MINIMUM ASSIGNMENT, which can be approximated within $v_1 K$ [10]. □

When $v_1 = 1$, then MIN-OWA $1|p_j = 1|\sum w_j U_j$ becomes MIN-MAX $1|p_j = 1|\sum w_j U_j$. Thus, the latter problem, under due date and weight uncertainty, admits a K-approximation algorithm.

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References

1. Aissi, H., Aloulou, M.A., Kovalyov, M.Y.: Minimizing the number of late jobs on a single machine under due date uncertainty. Journal of Scheduling **14**, 351–360 (2011)
2. Aloulou, M.A., Croce, F.D.: Complexity of single machine scheduling problems under scenario-based uncertainty. Operations Research Letters **36**, 338–342 (2008)
3. Averbakh, I.: On the complexity of a class of combinatorial optimization problems with uncertainty. Mathematical Programming **90**, 263–272 (2001)
4. Brucker, P.: Scheduling Algorithms, 5th edn. Springer Verlag, Heidelberg (2007)
5. Doerr, B.: Improved approximation algorithms for the Min-Max selecting Items problem. Information Processing Letters **113**, 747–749 (2013)
6. Garey, M.R., Johnson, D.S.: Computers and Intractability. A Guide to the Theory of NP-Completeness. W. H. Freeman and Company (1979)
7. Graham, R.L., Lawler, E.L., Lenstra, J.K., Rinnooy Kan, A.H.G.: Optimization and Approximation in Deterministic Sequencing and Scheduling: A Survey. Annals of Discrete Mathematics **5**, 169–231 (1979)
8. Karp, R.M.: Reducibility Among Combinatorial Problems. In: Complexity of Computer Computations, pp. 85–103 (1972)
9. Kasperski, A., Kurpisz, A., Zieliński, P.: Approximating the min-max (regret) selecting items problem. Information Processing Letters **113**, 23–29 (2013)
10. Kasperski, A., Zieliński, P.: Combinatorial optimization problems with uncertain costs and the OWA criterion. Theoretical Computer Science **565**, 102–112 (2015)
11. Kouvelis, P., Yu, G.: Robust Discrete Optimization and its applications. Kluwer Academic Publishers (1997)
12. Yager, R.R.: On ordered weighted averaging aggregation operators in multi-criteria decision making. IEEE Transactions on Systems, Man and Cybernetics **18**, 183–190 (1988)