Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

European Journal of Operational Research 158 (2004) 570–576

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/dsw

Discrete Optimization

The computational complexity of the relative robust shortest path problem with interval data ☆

Paweł Zieliński

Institute of Mathematics, Technical University of Opole, Luboszycka 3, 45-036 Opole, Poland

Received 29 August 2002; accepted 5 May 2003
Available online 12 August 2003

Abstract

The paper deals with the relative robust shortest path problem in a directed arc weighted graph, where arc lengths are specified as intervals containing possible realizations of arc lengths. The complexity status of this problem has been unknown in the literature. We show that the problem is $\mathcal{NP}$-hard.

Keywords: Combinatorial optimization; Robust optimization; Shortest path problem; Interval data; Computational complexity

1. Introduction

In this paper, we wish to investigate the computational complexity of the robust version of the shortest path problem, where arc lengths are specified as interval numbers. Each arc length can take on any value, any *realization*, in its interval. For this uncertainty representation the optimization criterion which is adopted is the *relative robustness* (*minimax regret*) *criterion*. This criterion is discussed in the book [4], which is entirely devoted to robust discrete optimization. One can find there a comprehensive treatment of the state of the art in robust discrete optimization, the application of this framework to several combinatorial problems and extensive references. Other robustness concepts were also studied by Mulvey et al. [7].

The problem, considered in this paper, consists in finding among all the paths the one, that over all the realizations of arc length intervals, minimizes the maximum deviation of the path length from the length of the shortest path in the corresponding realization, i.e. the path which has the best worst case performance. We refer to the problem as the relative robust shortest path problem.

Our study is not the first one. Kouvelis and Yu [4] (see also [9]) studied the relative robust shortest path with a discrete scenario set. Each *scenario* represents a possible realization of the arc lengths. They proved that the problem is $\mathcal{NP}$-hard with a bounded number of scenarios and is strongly $\mathcal{NP}$-hard with an unbounded number of scenarios. They conjectured that also the problem with interval data, considered here, is $\mathcal{NP}$-hard. Robust

☆ This work was supported by grant no. 7T11F02120 from the State Committee for Scientific Research (Komitet Badań Naukowych).

E-mail address: pziel@po.opole.pl (P. Zieliński).

0377-2217/$ - see front matter © 2003 Elsevier B.V. All rights reserved.
doi:10.1016/S0377-2217(03)00373-4

combinatorial optimization problems that are $\mathcal{NP}$-hard in the discrete scenario structure of uncertainty are usually conjectured intractable in the interval structure of uncertainty. However, for the problem of selecting p elements of minimum total weight out of a set of m elements with uncertainty in weights of the elements, the conjecture turned out to be false. Averbakh [1] proved that the problem is $\mathcal{NP}$-hard with a bounded number of scenarios but polynomially solvable in the interval case.

Unfortunately, in Karaşan et al. [3], Montemanni and Gambardella [5] and Montemanni et al. [6], which contain the same conjecture that the relative robust shortest path problem with interval data is intractable, no proof is given. Karaşan et al. [3] proposed a mixed integer programming approach with preprocessing for the problem. First, the mixed integer program is preprocessed by removing some arcs which will never be in an optimal path, and then it is solved by a standard software. In practice that approach is efficient only for acyclic layered graphs with a small width. A similar technique was applied for the solution of the relative robust spanning tree problem (see [8]). Montemanni et al. [6] provided a branch and bound algorithm for the relative robust shortest path problem. The algorithm is a refinement of the method presented in Montemanni and Gambardella [5].

In this paper we give a complete answer to the question about the complexity of the relative robust shortest path problem with interval data, namely that the problem is $\mathcal{NP}$-hard. We also show that the problem remains $\mathcal{NP}$-hard even when a graph is restricted to be directed acyclic planar and regular of degree three.

2. Problem definition

Let $G = (V, A)$ be a directed arc weighted graph with node set V, $|V| = n$, and arc set A, $|A| = m$. Two nodes $o \in V$ and $d \in V$ are distinguished as the *origin* and *destination* node, respectively. Lengths (weights) of the arcs are intervals $[\underline{l}_{ij}, \overline{l}_{ij}]$, $\underline{l}_{ij} \geqslant 0$, $(i,j) \in A$. The intervals express ranges of possible realizations of lengths. No probability distribution is assumed for arc lengths.

A realization of all arc lengths in G is called a *scenario s*, i.e. a length $l^s_{ij} \in [\underline{l}_{ij}, \overline{l}_{ij}]$ is assigned for each $(i,j) \in A$, where l^s_{ij} denotes the length of arc (i,j) in scenario s. Let P denote the set of all the paths in G from o to d. We denote the length of a path $p \in P$ in scenario s by $l^s_p = \sum_{(i,j)\in p} l^s_{ij}$. We use S to denote the set of possible scenarios.

Let us remind some notions concerning the robust shortest path problem (see, e.g., [3,4]).

Definition 1. The robust deviation for a path $p \in P$ in scenario s, denoted by d^s_p, is the difference between the length of p in s and the length of the shortest path $p^*(s) \in P$ in scenario s, i.e. $d^s_p = l^s_p - l^s_{p^*(s)}$.

Definition 2. A path $p^r \in P$ is a relative robust shortest path if it has the smallest (among all the paths from P) maximum (among all the scenarios from S) robust deviation, i.e. $p^r \in \arg\min_{p\in P} \times \max_{s\in S} (l^s_p - l^s_{p^*(s)})$.

Definition 3. For a given path $p \in P$, a scenario s_p which makes the robust deviation for p maximal is called a relative worst case scenario, i.e. $s_p \in \arg\max_{s\in S} d^s_p$.

The key statement for further considerations is Statement 1 which determines a relative worst case scenario s_p for a given path $p \in P$.

Statement 1 (Karaşan et al. [3]). For any $p \in P$, a relative worst case scenario s_p can be obtained as follows:

$$l^{s_p}_{ij} = \begin{cases} \overline{l}_{ij} & \text{if } (i,j) \in p, \\ \underline{l}_{ij} & \text{if } (i,j) \notin p. \end{cases}$$

In view of the above definitions we can define the relative robust shortest path problem (RRSPP for short) with arc length intervals: *find a relative robust shortest path in G.*

By Statement 1, one can equivalently define the problem RRSPP: *find $p \in P$ that minimizes $d^{s_p}_p$, where $d^{s_p}_p$ is the robust deviation for p in the relative worst case scenario s_p determined by means of Statement 1.*

The second formulation is more convenient for proving the complexity of RRSPP.

3. The complexity of RRSPP

In this section, we prove that RRSPP is $\mathcal{NP}$-hard by showing that a decision problem related to it is $\mathcal{NP}$-complete. The relative robust shortest path decision problem, Decision-RRSPP, is defined as follows:

INPUT: A directed graph $G = (V, A)$, weights (lengths) on the arcs $(i,j) \in A$ are determined by means of intervals $[\underline{l}_{ij}, \overline{l}_{ij}]$ (with integer bounds), $\underline{l}_{ij} \geqslant 0$, two nodes $o \in V$ and $d \in V$ are distinguished as the origin and destination node, respectively, and a nonnegative integer D.

QUESTION: Is there a path p from o to d in G such that $d_p^{s_p} \leqslant D$?

We show that Decision-RRSPP is $\mathcal{NP}$-complete by reducing a certain modified PARTITION problem, called MPARTITION, to it.

The MPARTITION problem is defined as follows:

INPUT: A finite set $\mathscr{A}$ of positive integers, $\mathscr{A} = \{a_1, \ldots, a_q\}$, having the overall sum of $2b$ and a positive integer $K < q$.

QUESTION: Is there a subset $\mathscr{A}' \subset \mathscr{A}$ that sums up exactly to b and $|\mathscr{A}'| = K$?

It is well known that MPARTITION is $\mathcal{NP}$-complete (see for instance [2] and comments on PARTITION given there).

Theorem 1. *Decision-RRSPP is $\mathcal{NP}$-complete.*

Proof. We claim that an instance of MPARTITION is polynomially reducible to an instance of Decision-RRSPP.

The reduction proceeds as follows. To each instance of MPARTITION, we associate a graph $G' = (V', A')$ with $4q+3$ nodes labelled $1, 2, \ldots, 4q+3$ (see Fig. 1). Node $2i$, $i = 1, \ldots, q$, is adjacent to nodes $2i-1$, $2i+1$ and $2(2q-i+2)$. Arcs $(2i-1, 2i)$, $(2i, 2i+1)$ and $(2i, 2(2q-i+2))$, $i = 1, \ldots, q$, have weight intervals $[0, M]$, $[M + a_i, M + a_i]$ and $[(q-i)M + \sum_{j=i}^{q} a_j, 4qM]$, respectively. M is a number such that $M > 2b$. Set $M = 2b+1$. The one-point intervals have been presented in Fig. 1 as precise numbers. Node $2(2q-i+2)$, $i = 1, \ldots, q$, is adjacent to nodes $2(2q-i+2)-1$ and $2(2q-i+2)+1$. Arcs $(2(2q-i+2)-1, 2(2q-i+2))$ and $(2(2q-i+2), 2(2q-i+2)+1)$, $i = 1, \ldots, q$, have weight intervals $[M + a_i, M + a_i]$ and $[0, M]$, respectively. Between node $2i-1$ and $2i+1$, and similarly, between $2(2q-i+2)-1$ and $2(2q-i+2)+1$ there are arcs $(2i-1, 2i+1)$ and $(2(2q-i+2)-1, 2(2q-i+2)+1)$ with weight interval $[0, 2M]$, for $i = 1, \ldots, q$. The arcs $(2q+1, 2q+2)$ and $(2q+2, 2q+3)$ both have weight interval $[0, 4qM]$. There is arc $(2q+1, 2q+3)$ having weight interval [0,0]. The arcs $(1, 2q+2)$ and $(2q+2, 4q+3)$ have weight intervals $[(q-K)M + b, 4qM]$ and $[KM + b, 4qM]$, respectively. This completes the definition of the graph G'. The construction of G' is done in time bounded by a polynomial in the size of MPARTITION. The rest of the input to Decision-RRSPP is defined as follows. Node $o \in V'$ is node 1, and $d \in V'$ is node $4q+3$ and the parameter D is set to be $3qM$.

We now prove that there exists a $p \in P$ in G' such that $d_p^{s_p} \leqslant 3qM$ if and only if there exists a subset $\mathscr{A}' \subset \mathscr{A}$ which sums up exactly to b and $|\mathscr{A}'| = K$.

$\Rightarrow$ Let $p \in P$ be a path such that $d_p^{s_p} \leqslant 3qM$. The scenario s_p is the relative worst case scenario for p determined by means of Statement 1. We determine a subset $\mathscr{A}' \subset \mathscr{A}$, $|\mathscr{A}'| = K$, and show that it sums up exactly to b.

First, we give two properties of the path p.

Property 1. *The path p must traverse arc $(2q+1, 2q+3)$.*

In order to prove Property 1, suppose to the contrary that p with $d_p^{s_p} \leqslant 3qM$ uses one of the parallel arcs to $(2q+1, 2q+3)$. Notice that these arcs have length intervals with upper bound equal to $4qM$. It is large enough to forbid path p to traverse these arcs. It is easily seen that each path from the set P traversing one of these parallel arcs

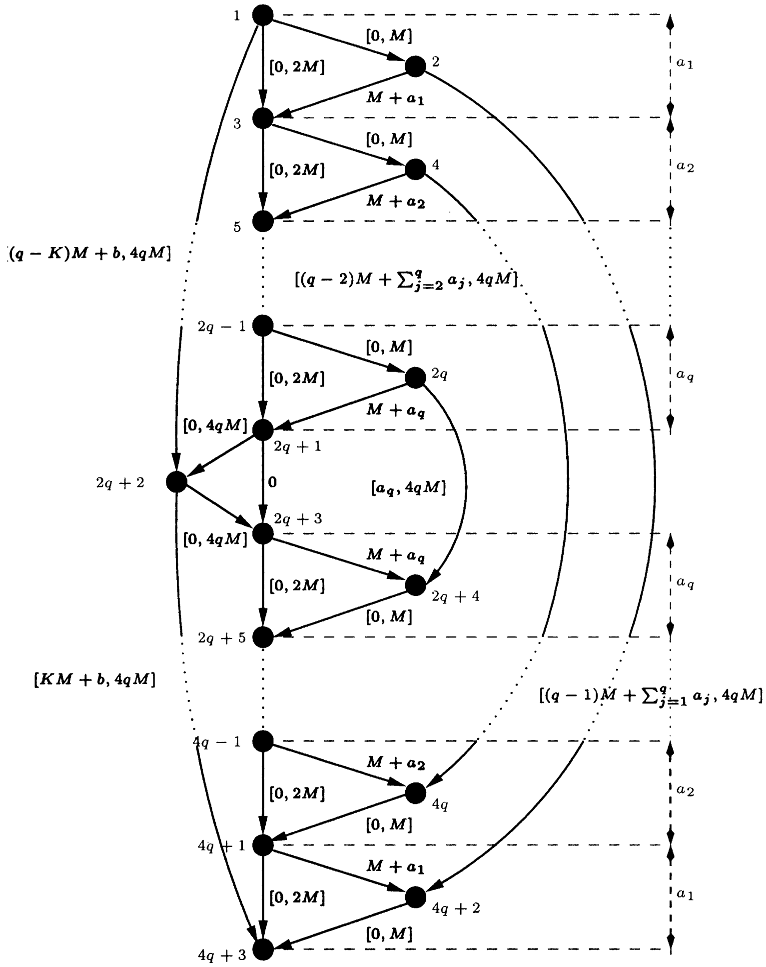

Fig. 1. The graph G'.

has the robust deviation in its relative worst case scenario of at least $4qM + 2M$. This contradicts our assumption.

Property 2. *The path p must use either arcs* $(2i-1, 2i)$, $(2i, 2i+1)$ (*the right portion of the triangle, which precedes arc* $(2q+1, 2q+3)$) *and arc* $(2(2q-i+2)-1, 2(2q-i+2)+1)$ (*the left portion of the triangle, which succeeds arc* $(2q+1, 2q+3)$) *or arc* $(2i-1, 2i+1)$ (*the left portion*) *and arcs* $(2(2q-i+2)-1, 2(2q-i+2))$, $(2(2q-i+2), 2(2q-i+2)+1)$ (*the right portion*), *for* $i = 1, \dots, q$.

To prove Property 2, assume to the contrary that there exist i's, $1 \leqslant i \leqslant q$, such that path p with $d_p^{s_p} \leqslant 3qM$ traverses either arcs $(2i-1, 2i+1)$ and $(2(2q-i+2)-1, 2(2q-i+2)+1)$ simultaneously, or $(2i-1, 2i)$, $(2i, 2i+1)$, $(2(2q-i+2)-1, 2(2q-i+2))$ and $(2(2q-i+2), 2(2q-i+2)+1)$ simultaneously.

Consider the case where there is no i such that path p uses arcs $(2i-1, 2i+1)$ and $(2(2q-i+2)-1, 2(2q-i+2)+1)$, simultaneously. Then there exists at least one i such that path p uses arcs $(2i-1, 2i)$, $(2i, 2i+1)$, $(2(2q-i+2)-1, 2(2q-i+2))$ and $(2(2q-i+2), 2(2q-i+2)+1)$, simultaneously. We choose a path $p' \in P$, that traverses arc $(2q+1, 2q+3)$ with the lengths of arcs on p' being at their lower bounds in s_p. This path has only one common arc with p, namely $(2q+1, 2q+3)$. The path p' can be determined uniquely. We immediately arrive to a contradiction, since $l_p^{s_p} > 4qM + 2b$ and $l_{p'}^{s_p} < qM + 2b$ and thus $d_p^{s_p} > 3qM$.

Consider the case when there exists at least one i such that path p uses arcs $(2i-1, 2i+1)$ and $(2(2q-i+2)-1, 2(2q-i+2)+1)$, simultaneously. Let us denote by i' the smallest such i. We choose a path $p' \in P$, that traverses arc $(2i', 2(2q-i'+2))$, parallel to arc $(2q+1, 2q+3)$, with the lengths of arcs on p' being at their lower bounds in s_p. The path p' is arc disjoint with path p and can be determined uniquely. Since $l_p^{s_p} \geqslant 4qM$ and $l_{p'}^{s_p} \leqslant (q-1)M + 2b$, the difference between $l_p^{s_p}$ and $l_{p'}^{s_p}$ is at least $3qM + M - 2b$. By definition, $2b < M$ and in consequence $d_p^{s_p} > 3qM$. This contradicts our assumption and the proof of Property 2 is complete.

We return to the main proof. From Properties 1 and 2, we deduce that the path p must traverse arc $(2q+1, 2q+3)$ and q times the right and q times the left portion of the triangles in G'. So, for scenario s_p, the path p is of length $4qM + 2b$. Since $d_p^{s_p} \leqslant 3qM$ and $l_p^{s_p} = 4qM + 2b$, the length of the shortest path, in s_p, is at least $qM + 2b$. A path from the set P that traverses arc $(2q+1, 2q+3)$ with the lengths of arcs on it being at their lower bounds in s_p is one of the paths of length equal exactly to $qM + 2b$. Let us denote it by $p^*(s_p)$. If p uses the right portion of the triangle, then $p^*(s_p)$ uses the left one. Similarly, if p uses the left portion of the triangle, then $p^*(s_p)$ uses the right one. The path $p^*(s_p)$ has only one common arc with p, namely $(2q+1, 2q+3)$. So, we obtain the following remark:

Remark 1. The $p^*(s_p)$ also satisfies Properties 1 and 2.

We show that the subpath of $p^*(s_p)$ from node 1 to $2q+1$ has length l' equal to $(q-K)M + b$. To do this, assume to the contrary that $l' < (q-K)M + b$. Therefore, the subpath of $p^*(s_p)$ from $2q+1$ to $4q+3$ is of length $l'' > KM + b$. This implies the existence of a path shorter than $p^*(s_p)$. It is composed of the subpath of $p^*(s_p)$ from 1 to $2q+1$ and arcs $(2q+1, 2q+2)$, $(2q+2, 4q+3)$. Likewise, assume that $l' > (q-K)M + b$. The subpath of $p^*(s_p)$ from $2q+1$ to $4q+3$ is of length $l'' < KM + b$. So, it follows that there exists a path shorter than $p^*(s_p)$. This path contains arcs $(1, 2q+2)$, $(2q+2, 2q+3)$ and the subpath of $p^*(s_p)$ from $2q+3$ to $4q+3$.

As path $p^*(s_p)$ has length $qM + 2b$, the subpath of $p^*(s_p)$ from node $2q+1$ to $4q+3$ is of length $KM + b$.

Let us determine a subset $\mathscr{A} \setminus \mathscr{A}'$. If the subpath of $p^*(s_p)$ from 1 to $2q+1$ traverses arc $(2i, 2i+1)$ (the right portion of the triangle), then a_i is included in $\mathscr{A} \setminus \mathscr{A}'$, $i = 1, \ldots, q$. Otherwise (it traverses $(2i-1, 2i+1)$) a_i is included in $\mathscr{A}'$. Since the subpath is of length $(q-K)M + b$, $M > \sum_{i=1}^{q} a_i$, and only arcs $(2i, 2i+1)$ have lengths $M + a_i$, $i = 1, \ldots, q$ (the rest of the arcs of the subpath are of length 0 in s_p), it must use $q-K$ times the right portion of the triangles and K times the left ones. This gives $|\mathscr{A} \setminus \mathscr{A}'| = q - K$ and $\sum_{\{i|a_i \in \mathscr{A} \setminus \mathscr{A}', 1 \leqslant i \leqslant q\}} a_i = b$. From this and the fact that $\mathscr{A}$ sums up to $2b$, we can conclude that $\sum_{\{i|a_i \in \mathscr{A}', 1 \leqslant i \leqslant q\}} a_i = b$ and $|\mathscr{A}'| = K$.

$\Leftarrow$ Assume that there exists a subset $\mathscr{A}' \subset \mathscr{A}$ that sums up exactly to b and $|\mathscr{A}'| = K$. We will construct a path $p \in P$ with $d_p^{s_p} = 3qM$.

Let us observe that each element $a_i \in \mathscr{A}$, $i = 1, \ldots, q$, corresponds to two triangles $(2i-1, 2i, 2i+1)$ and $(2(2q-i+2)-1, 2(2q-i+2), 2(2q-i+2)+1)$ linked by arc $(2i, 2(2q-i+2))$ (see Fig. 1). If $a_i \in \mathscr{A}'$, then we include arcs $(2i-1, 2i)$, $(2i, 2i+1)$ (the right portion of the first triangle) and $(2(2q-i+2)-1, 2(2q-i+2)+1)$ (the left portion of the second triangle) in the path p. Otherwise ($a_i \in \mathscr{A} \setminus \mathscr{A}'$), we include arcs $(2i-1, 2i+1)$ (the left portion of the first triangle) and $(2(2q-i+2)-1, 2(2q-i+2))$, $(2(2q-i+2), 2(2q-i+2)+1)$ (the right portion of the second triangle) in the path p. To complete p we add arc $(2q+1, 2q+3)$.

Consider scenario s_p determined according to Statement 1. Note that each element a_i, $i = 1, \ldots, q$, belongs either to $\mathscr{A}'$ or to $\mathscr{A} \setminus \mathscr{A}'$. So, the constructed path p uses the right portion of the first triangle and the left portion of the second one if $a_i \in \mathscr{A}'$ and the left portion of the first triangle and the right portion of the second one if $a_i \in \mathscr{A} \setminus \mathscr{A}'$ for each i. The subset $\mathscr{A}'$ sums up to b and $|\mathscr{A}'| = K$. Then the path p uses K times the right portions and $q - K$ times the left ones of the triangles, which precede arc $(2q+1, 2q+3)$. Hence, the length of the subpath of p from 1 to $2q+1$ is equal to $2qM + b$ in s_p. Similarly, as far as subset $\mathscr{A} \setminus \mathscr{A}'$ is concerned, which sums up exactly to b and $|\mathscr{A} \setminus \mathscr{A}'| = q - K$, the path p must use $q - K$ times the right portions and K times the left ones of the triangles, which succeed arc $(2q+1, 2q+3)$. The length of the subpath of p from $2q+3$ to $4q+3$ is equal to $2qM + b$ in s_p. Hence, length $l_p^{s_p} = 4qM + 2b$. The rest of the paths from set P in G' are of length at least $qM + 2b$ in s_p. In particular, paths whose arc lengths are at their lower bounds in s_p have lengths of exactly $qM + 2b$. Accordingly, the shortest path $p^*(s_p) \in P$ in s_p has length $qM + 2b$, i.e. $l_{p^*(s_p)}^{s_p} = qM + 2b$. Thus, $d_p^{s_p} = l_p^{s_p} - l_{p^*(s_p)}^{s_p} = 3qM$.

It remains to show that Decision-RRSPP is in $\mathscr{NP}$. A nondeterministic Turing machine first "guesses" a path p and then verifies p, i.e. it checks: whether p is a path from o to d in G and $d_p^{s_p} \leqslant D$ in the relative worst case scenario s_p determined as in Statement 1. The verification can be done in time bounded by a polynomial in the size of G as it amounts to solving the shortest path problem in G where the arc lengths are determined by scenario s_p. Thus, Decision-RRSPP is $\mathscr{NP}$-complete. □

From Theorem 1 we immediately obtain the computational complexity of RRSPP.

Corollary 1. *RRSPP is $\mathscr{NP}$-hard.*

The following theorem shows that planarity, acyclicness and bounded node degree (the degree of a node is the sum of the number of its incoming and outgoing arcs) are not sufficient to keep Decision-RRSPP from being $\mathscr{NP}$-complete.

Theorem 2. *Decision-RRSPP is $\mathscr{NP}$-complete, even if G is restricted to a planar acyclic graph with node degree three.*

Proof. It is clear that Decision-RRSPP is in $\mathscr{NP}$ for a planar acyclic digraph with node degree three.

We show that MPARTITION is polynomially reducible to Decision-RRSPP for a planar acyclic digraph with node degree three. The reduction is divided into two steps. In the first one, we construct exactly the same graph $G' = (V', A')$ as in the proof of Theorem 1. Note that G' is an acyclic planar digraph with the maximum node degree four. In the second step, we transform G' into a graph $G'' = (V'', A'')$ with the node degree three. To obtain G'', it is sufficient to split each node $k \in V'$ with degree four in G' by inserting an arc (k, k') having weight interval $[0, 0]$. It is evident that G'' is still acyclic, planar and each node has degree three. The construction of the above instance is done in time bounded by a polynomial in the size of MPARTITION.

What is left is to prove that G'' has a path $p \in P$ such that $d_p^{s_p} \leqslant 3qM$ if and only if there exists a subset $\mathscr{A}' \subset \mathscr{A}$ which sums up exactly to b and $|\mathscr{A}'| = K$. The proof of the above equivalence may be handled in the same way as for G' constructed in the proof of Theorem 1. This completes the proof. □

The following corollary is implied by Theorem 2.

Corollary 2. *RRSPP is $\mathscr{NP}$-hard, even if G is restricted to a planar acyclic graph with node degree three.*

4. Conclusions

In this paper, we have shown that the relative robust shortest path problem in a directed graph, where arc lengths are specified as intervals, is $\mathscr{NP}$-hard. We have also shown that the relative robust shortest path problem remains $\mathscr{NP}$-hard even for very restricted class of graphs, namely acyclic planar digraphs with node degree three. Thus, future considerations in this area would involve the development of efficient approximation algorithms and looking for polynomially solvable cases of this problem.

Acknowledgements

The author wishes to express his thanks to anonymous referees for careful readings of the paper and for several helpful comments and suggestions that improved the presentation.

References

[1] I. Averbakh, On the complexity of a class of combinatorial optimization problems with uncertainty, Mathematical Programming 25 (2001) 263–272.

[2] M.R. Garey, D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, W.H. Freeman and Co., San Francisco, 1979.

[3] O.E. Karaşan, M.C. Pinar, H. Yaman, The robust shortest path problem with interval data, submitted to Management Science, August 2001, Available from <http://ie.bilkent.edu.tr/~mustafap/pubs/rsp.ps>.

[4] P. Kouvelis, G. Yu, Robust Discrete Optimization and its Applications, Kluwer Academic Publishers, Boston, 1997.

[5] R. Montemanni, L.M Gambardella, An algorithm for the relative robust shortest path problem with interval data, Technical Report IDSIA-05-02, Istituto dalle Molle di Studi sull'Intelligenza Artificiale, February 2002, Available from <ftp://ftp.idsia.ch/pub/techrep/IDSIA-05-02.ps.gz>.

[6] R. Montemanni, L.M Gambardella, A.E. Rizzoli, A.V. Donati, A branch and bound algorithm for the robust shortest path problem with interval data, Technical Report IDSIA-09-02, Istituto dalle Molle di Studi sull'Intelligenza Artificiale, May 2002, Available from <ftp://ftp.idsia.ch/pub/techrep/IDSIA-09-02.ps.gz>.

[7] J.M. Mulvey, R.J. Vanderbei, S.A. Zenios, Robust optimization of large scale systems, Operations Research 43 (1995) 264–281.

[8] H. Yaman, O.E. Karaşan, M.C. Pinar, The robust spanning tree problem with interval data, Operations Research Letters 29 (2001) 31–40.

[9] G. Yu, J. Yang, On the robust shortest path problem, Computers and Operations Research 25 (1998) 457–468.