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Ranking fuzzy interval numbers in the setting of random sets – further results

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Abstract

We present some new properties of several fuzzy order relations, defined on the set of fuzzy numbers, from among those introduced in [S. Chanas, M. Delgado, J.L. Verdegay, M.A. Vila, Information Sciences 69 (1993) 201–217]. The main result is proving that four from among the relations considered in [S. Chanas, M. Delgado, J.L. Verdegay, M.A. Vila, Information Sciences 69 (1993) 201–217] are strongly transitive (s-transitive).

1. Introduction

In paper [1] the authors define eight fuzzy relations defined on the set of fuzzy numbers, making use of some relationships between fuzzy and random sets (see [4]). They prove a number of properties of those relations. Among other things, they show that three from among them are max–min transitive. In this paper we prove further properties of some of the relations introduced in [1]. The main result is showing that four from among them are strongly transitive (s-transitive). The property of s-transitiveness is of great importance in case the relations are applied to the decision making according to Orlovsky's concept (see [6]). At the end of this paper we present shortly Orlovsky's concept of decision making with fuzzy preference relation, pointing out at the same

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time what significance for the concept has the property of s-transitiveness of the preference relation used there.

All the relations properties presented here are proven on the assumption that their domain is limited to the set of fuzzy numbers of the same $L-R$ type. This set is closed with respect to the operation of addition and multiplication by a non-negative parameter. Such a limitation was not needed in [1]. The properties proven there are true also on the assumption that fuzzy numbers being compared with respect to the given relation are of different types.

2. Fuzzy order relations in the set of fuzzy numbers and their properties

Let a set of interval fuzzy numbers of the same $L-R$ type be given. We will denote this set by $FN(L,R)$. The shape functions L and R can be arbitrary, but fixed. Let us recall that a fuzzy number A is an interval fuzzy number of the type $L-R$, $A \in FN(L,R)$, if and only if its membership function, $\mu_A : \mathbb{R} \to [0,1]$, has the form (see [3]):

$$\mu_A(x) = \begin{cases} 1 & \text{for } x \in [\underline{a}, \overline{a}], \\ L\left(\frac{\underline{a}-x}{\alpha}\right) & \text{for } x \leqslant \underline{a}, \\ R\left(\frac{x-\overline{a}}{\beta}\right) & \text{for } x \geqslant \overline{a}, \end{cases} \tag{1}$$

where L and R are non-negative functions defined on the half-line $[0,\infty)$, non-increasing and such that $L(0) = R(0) = 1$. A fuzzy number $A \in FN(L,R)$ with a membership function (1) is conventionally denoted as

$$A = (\underline{a}, \overline{a}, \alpha_A, \beta_A)_{L-R}.$$

In the following we also assume that shape functions L and R are continuous and strictly decreasing on those parts of the half-line $[0,\infty)$ where they are positive. Here are some examples of functions which can be used as shape functions: $\max(0, 1-y)$, e^{-y^p}, $\max(0, 1-y^p)$, $y \in [0,\infty)$, $p \geqslant 1$.

Further on we will analyze six fuzzy relations R_i, $i = 1,\ldots,6$, defined on set $FN(L,R)$. The membership functions of these relations $\mu_i : FN(L,R)^2 \to [0,1]$, $i = 1,\ldots,6$, are given by the formulae:

$$\mu_1(A,B) = \mathrm{Prob}\{\underline{a} - L^{-1}(Y)\alpha_A > \overline{b} + R^{-1}(Y)\beta_B\}, \tag{2}$$

$$\mu_2(A,B) = \mathrm{Prob}\{\underline{a} - L^{-1}(Y)\alpha_A > \overline{b} + R^{-1}(Z)\beta_B\}, \tag{3}$$

$$\mu_3(A,B) = \mathrm{Prob}\{\underline{a} - L^{-1}(Y)\alpha_A \geqslant \underline{b} - L^{-1}(Y)\alpha_B\}, \tag{4}$$

$$\mu_4(A,B) = \mathrm{Prob}\{\overline{a} + R^{-1}(Y)\beta_A > \overline{b} + R^{-1}(Y)\beta_B\}, \tag{5}$$

$$\mu_5(A,B) = \mathrm{Prob}\{\overline{a} + R^{-1}(Y)\beta_A \geqslant \underline{b} - L^{-1}(Y)\alpha_B\}, \tag{6}$$

$$\mu_6(A,B) = \mathrm{Prob}\{\overline{a} + R^{-1}(Z)\beta_A \geqslant \underline{b} - L^{-1}(Y)\alpha_B\}, \tag{7}$$

where Y and Z are independent random variables with the uniform distribution on interval $[0, 1]$.

In order to simplify the notation, the symbols standing for the relations analyzed here have been changed with respect to those used in [1]. The relations correspond to the following six ones from among eight relations considered in [1]: $R_1 \to R_{11}$, $R_2 \to R_{13}$, $R_3 \to R_{21}$, $R_4 \to R_{31}$, $R_5 \to R_{41}$, $R_6 \to R_{43}$.

In the following eight lemmas we formulate a number of properties of relations R_i, $i = 1, \ldots, 6$. Some of these properties will be used in the proofs of three further theorems concerning the s-transitiveness of those relations. However, these properties are of a wider importance and can also be used for other purposes.

Lemma 1. *Let $A, B \in FN(L, R)$ be two arbitrary fuzzy numbers of the type $L - R$. For both fuzzy relations R_i, $i = 1, 2$ the following condition is fulfilled*:

$$\mu_i(A, B) > 0 \Rightarrow \underline{a} > \overline{b} \text{ (or equivalently } \underline{a} \leqslant \overline{b} \Rightarrow \mu_i(A, B) = 0). \quad (8)$$

Proof. Obvious. Condition (8) follows directly from formulae (2) and (3) as well as from the properties of the shape function of a fuzzy number (indeed, it holds $L^{-1}(Y) \geqslant 0$ and $R^{-1}(Y) \geqslant 0$). □

The following lemma is a consequence of Lemma 1:

Lemma 2. *Relation R_i, $i = 1, 2$ is antisymmetric, i.e. for any two fuzzy numbers $A, B \in FN(L, R)$ the following condition is fulfilled*:

$$\mu_i(A, B) > 0 \Rightarrow \mu_i(B, A) = 0. \quad (9)$$

Proof. Let $\mu_i(A, B) > 0$, $i = 1, 2$. From Lemma 1 we get $\underline{a} > \overline{b}$. From this inequality and from the properties of fuzzy numbers (it holds $\underline{a} \leqslant \overline{a}$ and $\underline{b} \leqslant \overline{b}$) we obtain the inequality $\underline{b} \leqslant \overline{a}$, from which – after applying Lemma 1 again – we arrive at the equality $\mu_i(B, A) = 0$. □

Lemma 3. *For any two fuzzy numbers $A, B \in FN(L, R)$ the following conditions are fulfilled*:

$$\underline{a} - \underline{b} - L^{-1}\left(\frac{1}{2}\right)(\alpha_A - \alpha_B) \geqslant 0 \iff \mu_3(A, B) \geqslant \frac{1}{2}, \quad (10)$$

$$\underline{a} - \underline{b} - L^{-1}\left(\frac{1}{2}\right)(\alpha_A - \alpha_B) > 0 \Rightarrow \mu_3(A, B) > \frac{1}{2}. \quad (11)$$

Proof. Making use of formula (4) and of the fact that L^{-1} is a strictly decreasing function on the interval [0,1], the following formula for calculating the value of the membership function $\mu_3(A, B)$ can be derived:

$$\mu_3(A,B) = \begin{cases} L\left(\frac{\underline{a}-\underline{b}}{\alpha_A-\alpha_B}\right) & \text{for } \alpha_A < \alpha_B \text{ and } \underline{a} < \underline{b}, \\ 1 & \text{for } \alpha_A \leqslant \alpha_B \text{ and } \underline{a} \geqslant \underline{b}, \\ 1 - L\left(\frac{\underline{a}-\underline{b}}{\alpha_A-\alpha_B}\right) & \text{for } \alpha_A > \alpha_B \text{ and } \underline{a} \geqslant \underline{b}, \\ 0 & \text{for } \alpha_A \geqslant \alpha_B \text{ and } \underline{a} < \underline{b}. \end{cases} \tag{12}$$

Formula (12) allows an easy justification of conditions (10) and (11). □

Remark 1. *Implication* (11) *is fulfilled also in the other direction, but only for fuzzy numbers* $A, B \in FN(L,R)$ *fulfilling the condition* $\underline{a} \neq \underline{b}$ *or* $\alpha_A \neq \alpha_B$. *In the case* $\underline{a} = \underline{b}$ *and* $\alpha_A = \alpha_B$ *we have* $\mu_3(A,B) = 1 > \frac{1}{2}$ *with* $\underline{a} - \underline{b} - L^{-1}\left(\frac{1}{2}\right)(\alpha_A - \alpha_B) = 0$.

Using formula (12) again, it is easy to prove the following lemma:

Lemma 4. *For any two fuzzy numbers* $A, B \in FN(L,R)$ *such that* $\underline{a} \neq \underline{b}$ or $\alpha_A \neq \alpha_B$ *the following condition is fulfilled*:

$$\mu_3(A,B) + \mu_3(B,A) = 1. \tag{13}$$

Two further lemmas concern relation R_4.

Lemma 5. *For any two fuzzy numbers* $A, B \in FN(L,R)$ *the following conditions are fulfilled*:

$$\overline{a} - \overline{b} + R^{-1}\left(\frac{1}{2}\right)(\beta_A - \beta_B) \geqslant 0 \iff \mu_4(A,B) \geqslant \frac{1}{2}, \tag{14}$$

$$\overline{a} - \overline{b} + R^{-1}\left(\frac{1}{2}\right)(\beta_A - \beta_B) > 0 \iff \mu_4(A,B) > \frac{1}{2}. \tag{15}$$

Proof. Making use of formula (5) and of the properties of the shape function R^{-1}, it is possible to derive a formula, similar to that numbered (12), for the value of the membership function μ_4:

$$\mu_4(A,B) = \begin{cases} R\left(\frac{\overline{b}-\overline{a}}{\beta_A-\beta_B}\right) & \text{for } \beta_A > \beta_B \text{ and } \overline{a} \leqslant \overline{b}, \\ 1 & \text{for } \beta_A \geqslant \beta_B \text{ and } \overline{a} > \overline{b}, \\ 1 - R\left(\frac{\overline{b}-\overline{a}}{\beta_A-\beta_B}\right) & \text{for } \beta_A < \beta_B \text{ and } \overline{a} > \overline{b}, \\ 0 & \text{for } \beta_A \leqslant \beta_B \text{ and } \overline{a} \leqslant \overline{b}. \end{cases} \tag{16}$$

Making use of (16), it is easy to justify conditions (14) and (15). □

Taking into account formula (16) again, it is possible to prove the following lemma:

Lemma 6. *For any two fuzzy numbers* $A, B \in FN(L, R)$ *such that* $\beta_A \neq \beta_B$ *or* $\overline{a} \neq \overline{b}$ *the following condition holds*:

$$\mu_4(A, B) + \mu_4(B, A) = 1. \tag{17}$$

Lemma 7. *Let* $A, B \in FN(L, R)$ *be any fuzzy numbers of the* $L - R$ *type. For both fuzzy relations* R_i, $i = 5, 6$, *the following condition is fulfilled*:

$$\mu_i(A, B) < 1 \iff \overline{a} < \underline{b} \quad (\text{or equivalently } \overline{a} \geqslant \underline{b} \iff \mu_i(A, B) = 1). \tag{18}$$

Proof. Obvious. Condition (18) follows directly from formulae (6) and (7) as well as from the properties of the shape function ($L^{-1}(Y) \geqslant 0$ and $R^{-1}(Y) \geqslant 0$). □

Lemma 8. *Both relations* R_i, $i = 5, 6$ *satisfy the condition*

$$\mu_i(A, B) < 1 \Rightarrow \mu_i(B, A) = 1. \tag{19}$$

Proof. Let us suppose that $\mu_i(A, B) < 1$. From Lemma 7 we get the condition $\overline{a} < \underline{b}$. From this condition and from the properties of fuzzy numbers (it holds $\underline{a} \leqslant \overline{a}$ and $\underline{b} \leqslant \overline{b}$) we obtain the inequality $\overline{b} > \underline{a}$, from which – using Lemma 7 again – we conclude the inequality $\mu_i(B, A) = 1$. □

Let us now remind the notion of s-transitiveness introduced by Kołodziejczyk in [5].

Definition 1. A fuzzy relation $\mathscr{R}$, defined on a set $X \times X$, is called s-transitive if for any x, y, $z \in X$ inequalities

$$\mu_{\mathscr{R}}(x, y) > \mu_{\mathscr{R}}(y, x) \text{ and } \mu_{\mathscr{R}}(y, z) > \mu_{\mathscr{R}}(z, y)$$

imply inequality

$$\mu_{\mathscr{R}}(x, z) > \mu_{\mathscr{R}}(z, x).$$

Remark 2. *The notion of s-transitiveness is more general than that of max–min transitiveness, i.e. the category of max–min transitive relations is included in that of s-transitive relations* (*see* [5]).

For instance, relations R_i, $i = 3, \ldots, 6$ are s-transitive, but are not max–min transitive. Examples showing that they are not max–min transitive are constructed in [1].

Since, as it is shown in [1], relations R_1 and R_2 are max–min transitive, according to the above remark they are also s-transitive.

We will show now that also fuzzy relations R_i, $i = 3, \ldots, 6$ are s-transitive.

Theorem 1. *Relation R_3 is s-transitive.*

Proof. Let $A, B, C \in FN(L, R)$ be any fuzzy numbers of the $L-R$ type ($A \neq B$, $B \neq C$, $A \neq C$). Let us suppose that

$$\mu_3(A,B) > \mu_3(B,A) \quad \text{and} \quad \mu_3(B,C) > \mu_3(C,B). \tag{20}$$

It is to show that $\mu_3(A,C) > \mu_3(C,A)$.

From assumptions (20) and the properties of the membership function of a fuzzy relation ($\mu_3(A,B) \leqslant 1$, $\mu_3(B,C) \leqslant 1$) follow the following inequalities:

$$\mu_3(B,A) < 1 \quad \text{and} \quad \mu_3(C,B) < 1. \tag{21}$$

On the basis of inequality (21) and formula (4) (or (12)) we conclude that the following conditions are true: $\underline{a} \neq \underline{b}$ or $\alpha_A \neq \alpha_B$, $\underline{b} \neq \underline{c}$ or $\alpha_B \neq \alpha_C$. From this as well as from Lemma 4 and condition (20) we obtain the following inequalities:

$$\mu_3(A,B) > \frac{1}{2} \quad \text{and} \quad \mu_3(B,C) > \frac{1}{2}. \tag{22}$$

Making use of inequality (22) and of Remark 1 we conclude that

$$\underline{a} - \underline{b} - L^{-1}\left(\frac{1}{2}\right)(\alpha_A - \alpha_B) > 0, \tag{23}$$

$$\underline{b} - \underline{c} - L^{-1}\left(\frac{1}{2}\right)(\alpha_B - \alpha_C) > 0. \tag{24}$$

Summing inequalities (23) and (24) by sides, we obtain

$$\underline{a} - \underline{c} - L^{-1}\left(\frac{1}{2}\right)(\alpha_A - \alpha_C) > 0. \tag{25}$$

From (25) it follows that the condition $\underline{a} \neq \underline{c}$ or $\alpha_A \neq \alpha_C$ is fulfilled. From this as well as from condition (25) and Lemma 3 it follows that

$$\mu_3(A,C) > \frac{1}{2}. \tag{26}$$

Making use of Lemma 4 and inequality (26), we get

$$\mu_3(A,C) > \mu_3(C,A),$$

which proves the theorem. □

Theorem 2. *Relation R_4 is s-transitive.*

Proof. Let $A, B, C \in FN(L, R)$ be any fuzzy numbers of the $L-R$ type ($A \neq B$, $B \neq C$, $A \neq C$). Let us suppose that

$$\mu_4(A,B) > \mu_4(B,A) \quad \text{and} \quad \mu_4(B,C) > \mu_4(C,B). \tag{27}$$

It is to show that $\mu_4(A,C) > \mu_4(C,A)$.

From assumptions (27) and the properties of the membership function of a fuzzy relation ($\mu_4(B,A) \geqslant 0$, $\mu_4(C,B) \geqslant 0$) follow the following inequalities:

$$\mu_4(A,B) > 0 \quad \text{and} \quad \mu_4(B,C) > 0. \tag{28}$$

On the basis of inequality (28) and formula (5) (or (16)) we conclude that the following conditions are true: $\overline{a} \neq \overline{b}$ or $\beta_A \neq \beta_B$, $\overline{b} \neq \overline{c}$ or $\beta_B \neq \beta_C$. From this as well as from Lemma 6 and condition (27) we obtain the following inequalities:

$$\mu_4(A,B) > \frac{1}{2} \quad \text{and} \quad \mu_4(B,C) > \frac{1}{2}. \tag{29}$$

Making use of inequality (29) and of Lemma 5 we conclude that

$$\overline{a} - \overline{b} + R^{-1}\left(\frac{1}{2}\right)(\beta_A - \beta_B) > 0, \tag{30}$$

$$\overline{b} - \overline{c} + R^{-1}\left(\frac{1}{2}\right)(\beta_B - \beta_C) > 0. \tag{31}$$

Summing inequalities (30) and (31) by sides, we obtain

$$\overline{a} - \overline{c} + R^{-1}\left(\frac{1}{2}\right)(\beta_A - \beta_C) > 0. \tag{32}$$

From (32) it follows that the condition $\overline{a} \neq \overline{c}$ or $\beta_A \neq \beta_C$ is fulfilled. From this as well as from condition (32) and Lemma 5 it follows that

$$\mu_4(A,C) > \frac{1}{2}. \tag{33}$$

Making use of Lemma 6 and inequality (33), we get

$$\mu_4(A,C) > \mu_4(C,A),$$

which proves the theorem. □

Theorem 3. *Both relations R_i, $i = 5, 6$, are s-transitive.*

Proof. Let $A, B, C \in FN(L,R)$ be any fuzzy numbers of the $L-R$ type ($A \neq B$, $B \neq C$, $A \neq C$). Let us suppose that

$$\mu_i(A,B) > \mu_i(B,A) \quad \text{and} \quad \mu_i(B,C) > \mu_i(C,B). \tag{34}$$

It is to show that $\mu_i(A,C) > \mu_i(C,A)$.

From assumptions (34) and the properties of the membership function of a fuzzy relation ($\mu_i(A,B) \leqslant 1$, $\mu_i(B,C) \leqslant 1$) follow the following inequalities:

$$\mu_i(B,A) < 1 \quad \text{and} \quad \mu_i(C,B) < 1. \tag{35}$$

From (35) and Lemma 7 we have

$$\underline{a} > \overline{b} \;\text{ and }\; \underline{b} > \overline{c}. \tag{36}$$

Making use of (36) and of the fact that $\underline{b} \leqslant \overline{b}$ we conclude

$$\underline{a} > \overline{c}. \tag{37}$$

From inequality (37) and Lemma 7 it follows that

$$\mu_i(C, A) < 1. \tag{38}$$

From (38) and Lemma 8 we have $\mu_i(A, C) = 1$. From this and condition (38) we get the inequality

$$\mu_i(A, C) > \mu_i(C, A),$$

which proves the theorem. □

3. Decision making with a fuzzy preference relation

We will now shortly present Orlovsky's concept of an optimal decision selection on the basis of a fuzzy preference relation defined on the set of feasible solutions [6]. Let a set X of alternatives be given. Let an objective function $f : X \to Z$ be defined on this set, where Z is the evaluation space. Usually Z is equal to the set of real numbers (exact evaluations). However, it can also be a set of fuzzy numbers, e.g. $Z = FN(L, R)$, (imprecise evaluations of the decision utility), or a set whose elements are not numbers. Let us suppose that on set Z there is defined a fuzzy preference relation $\mathscr{R}$ with the membership function $\mu_{\mathscr{R}} : Z \times Z \to [0, 1]$. The value $\mu_{\mathscr{R}}(z_1, z_2)$ gives the degree to which evaluation $z_1 \in Z$ is preferred by the decision maker with respect to evaluation $z_2 \in Z$, i.e. to which degree the relation $z_1 \geqslant z_2$ is true. Relation $\mathscr{R}$ defines a fuzzy order on the alternatives set X. The value $\mu_{\mathscr{R}}(f(x_1), f(x_2))$, $x_1, x_2 \in X$, gives the degree to which alternative $x_1 \in X$ is better than alternative $x_2 \in X$ with respect to objective function f and the preference relation $\mathscr{R}$.

Definition 2. The set of non-dominated solutions in X with respect to fuzzy relation $\mathscr{R}$, denoted with ND, is the fuzzy set in X with the membership function

$$\mu_{\mathrm{ND}}(x) = \inf_{y \in X}[1 - \mu_{\mathscr{R}^s}(f(y), f(x))] = 1 - \sup_{y \in X} \mu_{\mathscr{R}^s}(f(y), f(x)), \quad x \in X,$$

where $\mathscr{R}^s$ is a fuzzy relation of strict preference connected to relation $\mathscr{R}$. According to Orlovsky's concept, the membership function of relation $\mathscr{R}^s$ has the following form:

$$\mu_{\mathscr{R}^s}(z_1, z_2) = \max\{0, \mu_{\mathscr{R}}(z_1, z_2) - \mu_{\mathscr{R}}(z_2, z_1)\}.$$

The value of membership function $\mu_{\mathrm{ND}}(x)$ stands for the degree to which alternative $x \in X$ is not dominated by another alternative from set X (the degree of truth of the statement "x is not dominated by any alternative $y \in X$").

The best alternative seems to be the one which to the maximal degree is non-dominated by other alternatives, i.e. one fulfilling the condition:

$$\mu_{\mathrm{ND}}(x) = \max_{y \in X} \mu_{\mathrm{ND}}(y).$$

If $\mu_{\mathrm{ND}}(x) = 1$ then alternative x is called an unfuzzy non-dominated solution. The set of all the unfuzzy non-dominated solutions is denoted with UND.

Unfuzzy non-dominated solutions – if there are any, i.e. if UND$\neq \emptyset$ – seem to be of great practical importance. It is so, because with respect to them there is no doubt that they are the best ones: the statement that they are not dominated by any other solution from the set of feasible solutions X is true to degree 1. It is thus interesting to ask on which conditions set UND is not empty.

In his source paper [6] Orlovsky has shown that if fuzzy preference relation $\mathcal{R}$ is max–min transitive then UND$\neq \emptyset$. Kołodziejczyk has given in [5] more general condition, proving that UND $\neq \emptyset$ if relation $\mathcal{R}$ is s-transitive.

In [2] the authors of the present paper analyze the linear programming problem in which the objective function coefficients are given in the imprecise form of fuzzy numbers of the $L-R$ type, i.e. the following one:

$$F(x) = \sum_{j=1}^{n} C_j x_j \rightarrow \max, \tag{39}$$

$$x \in X: \quad \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leqslant b_i, & i = 1, \ldots, m, \\ x_j \geqslant 0, & j = 1, \ldots, n, \end{cases}$$

where $C_j = (\underline{c}_j, \overline{c}_j, \alpha_j, \beta_j)_{L-R}$, $j = 1, \ldots, n$ are fuzzy numbers of an identical $L-R$ type. The other coefficients of the problem are crisp.

The evaluation of a solution x, $F(x)$, is in this situation also a fuzzy number of the $L-R$ type. This means that the space of the evaluations of the feasible solutions of problem (39) is equal to set $FN(L,R)$. Making use of the fact that relations R_i, $i = 1, \ldots, 6$, considered here, are s-transitive, and thus the set UND is not empty with respect to each one of them, the authors of paper [2] undertook an analysis of problem (39) from the angle of a computationally effective characterization of set UND for all the relations. They show that for each relation R_i, $i = 1, \ldots, 6$, set UND for problem (39) is equal either to the set of solutions of a certain system of linear inequalities or to the set of solutions of a certain classical linear programming problem. This means that fuzzy non-dominated solutions of problem (39), with respect to the above mentioned relations, can be effectively determined by means of well developed, classical linear programming methods. An interested reader can consult source paper [2].

4. Conclusions

In the paper we prove further useful properties of fuzzy preference relations defined on the set of fuzzy numbers, which were introduced in [1]. One of the most important properties is that of s-transitiveness, which can be used in decision making. In order to point out the usefulness in decision making of the property of s-transitiveness of a relation, in the last part of the paper we present Orlovsky's concept of decision selection with respect to a fuzzy preference relation, indicating the point in which the property in question is of a substantial practical importance for this concept.

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