

**From the strict Chebyshev approximant of a vector  
to the strict spectral approximant of a matrix**

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**Abstract.** The main aim of this review paper is approximation of a complex rectangular matrix, with respect to the unitarily invariant norms, by matrices from a linear subspace or from a convex closed subset of matrices. In particular, we focus on the properties and characterizations of the strict spectral approximant which is the best in some sense among all approximants to a given matrix with respect to the spectral norm. We formulate a conjecture that the strict spectral approximant to a matrix by matrices from a linear subspace is the limit of approximants with respect to the  $c_p$  norms of Schatten when  $p$  tends to infinity. This conjecture corresponds to the conjecture stated by Rogers and Ward on approximation by positive semidefinite matrices and to the known analogous property of the strict Chebyshev approximation of a vector, proved by Descoux. We describe some special cases of an approximation of a matrix that confirm our conjecture and we discuss an attempt to prove the conjecture.

Additionally, we focus on orthogonality of matrices in the sense of Birkhoff and James and approximation by matrices whose spectrum is in a strip. For the last case we recall a conjecture on characterization of the best approximation of a matrix in the spectral norm by matrices whose spectrum is in a strip. This kind of approximation is a generalization of the famous concept of Halmos of approximation by positive operators.

For a convenience of a reader we recall properties of the Chebyshev solutions of overdetermined system of linear equations and the solutions with respect to the  $l_p$  norm, and the properties of dual matrices, related to subdifferentials of norm of a matrix, defined for the unitarily invariant norms.

The paper is based on a series of papers by the author and on many papers of other authors, related to the considered problems.

**Keywords:** approximation of matrices, unitarily invariant norm,  $c_p$  norm of Schatten, lexicographic ordering, subdifferential of norm, dual matrix, strict spectral approximant,  $c_p$  minimal approximation, Birkhoff-James orthogonality, zero trace matrix, positive definite matrix, restricted spectrum, overdetermined system of linear equations,  $l_p$  solution, Chebyshev solution, strict Chebyshev solution, Pólya algorithm.